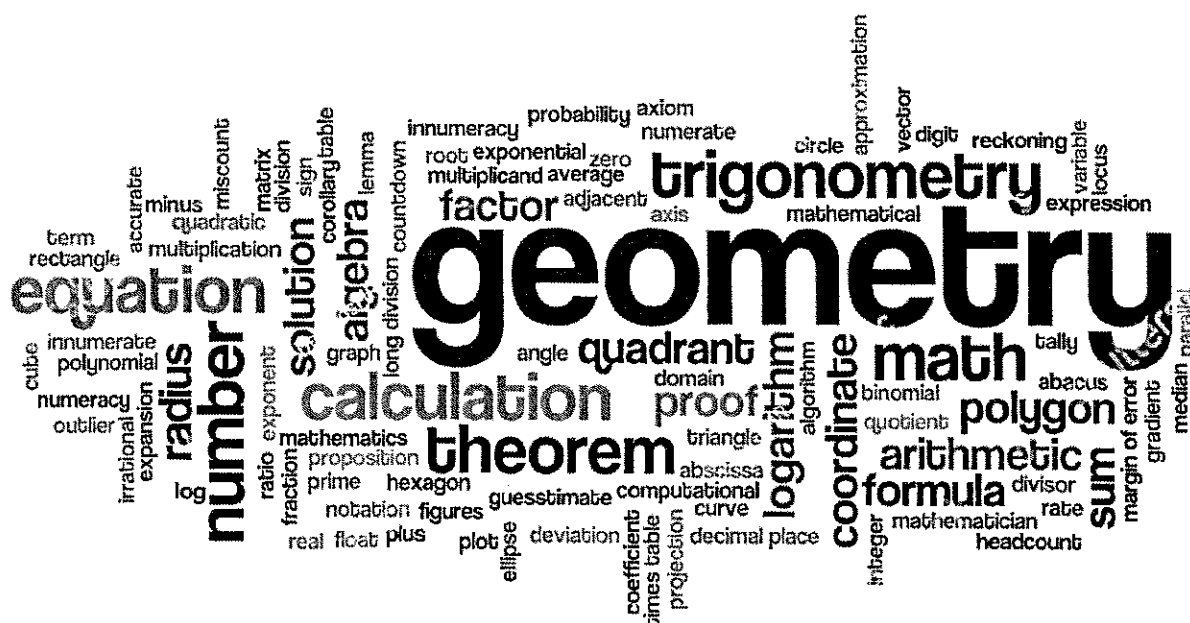


Transformations in the Coordinate Plane

<u>Title</u>	<u>Page #</u>
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This Packet Belongs To:



Below are the formulas you may find useful as you take the test. However, you may find that you do not need to use all of the formulas. You may refer to this formula sheet as often as needed.

Geometry Formulas

Perimeter

The perimeter of a polygon is equal to the sum of the length of its sides.

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Coordinates of point which partitions a directed line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

$$(x, y) = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a} \right)$$

OR

$$(x, y) = \left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1) \right)$$

Circumference of a Circle

$$C = \pi d \text{ or } C = 2\pi r$$

$$\pi \approx 3.14$$

Arc Length of a Circle

$$\text{Arc Length} = \frac{2\pi r\theta}{360}$$

Area

Triangle $A = \frac{1}{2}bh$

Rectangle $A = bh$

Circle $A = \pi r^2$

Area of a Sector of a Circle

$$\text{Area of Sector} = \frac{\pi r^2 \theta}{360}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Trigonometric Relationships

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}; \cos \theta = \frac{\text{adj}}{\text{hyp}}; \tan \theta = \frac{\text{opp}}{\text{adj}}$$

Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Volume

Cylinder $V = \pi r^2 h$

Pyramid $V = \frac{1}{3}Bh$

Cone $V = \frac{1}{3}\pi r^2 h$

Sphere $V = \frac{4}{3}\pi r^3$

Statistics Formulas

Conditional Probability

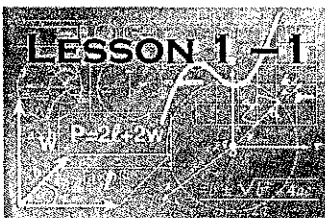
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

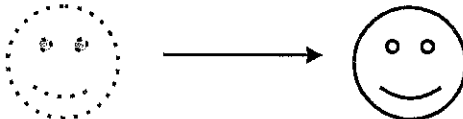
Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Introduction to Transformations and Rotations

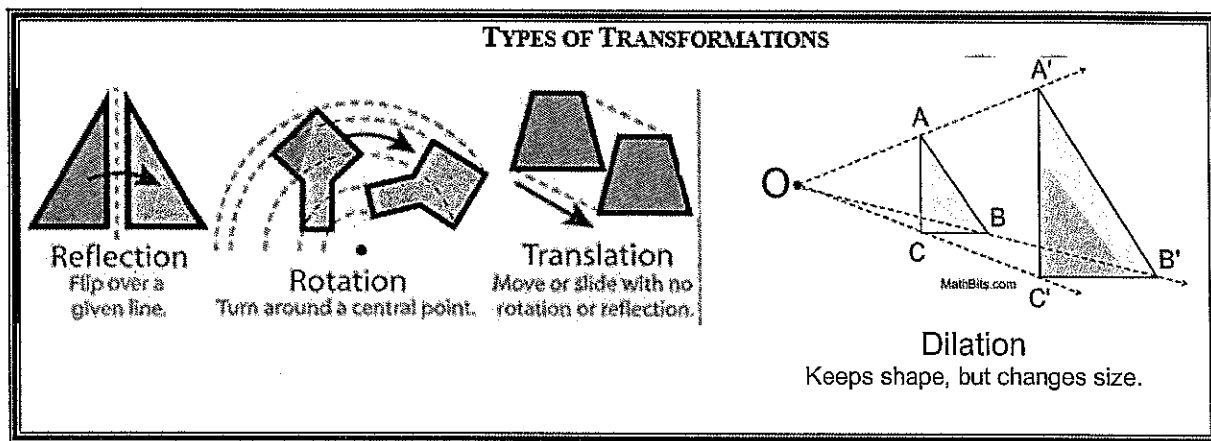
Transformation: maps, or moves, the _____ (original figure) onto the _____ (new figure).



Vocabulary:

- Angle of Rotation
- Image
- Pre – image
- Rotation
- Transformation

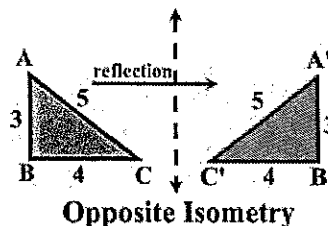
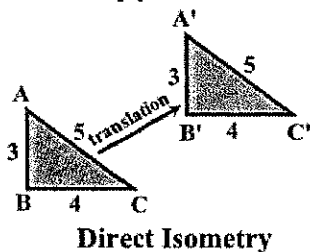
Naming Transformations:



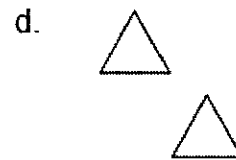
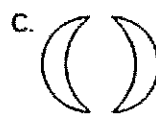
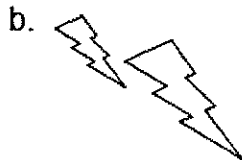
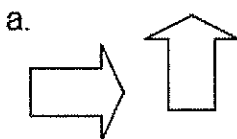
Some transformations preserve size and shape. Translations and rotations are isometric transformations.

Rigid Motion (Isometry):

- **Direct isometry** _____ orientation and order.
- **Opposite isometry** _____ the order, but _____ the orientation.



You Try: Label the following as a Reflection, Rotation, Translation or Dilation.



Rotations

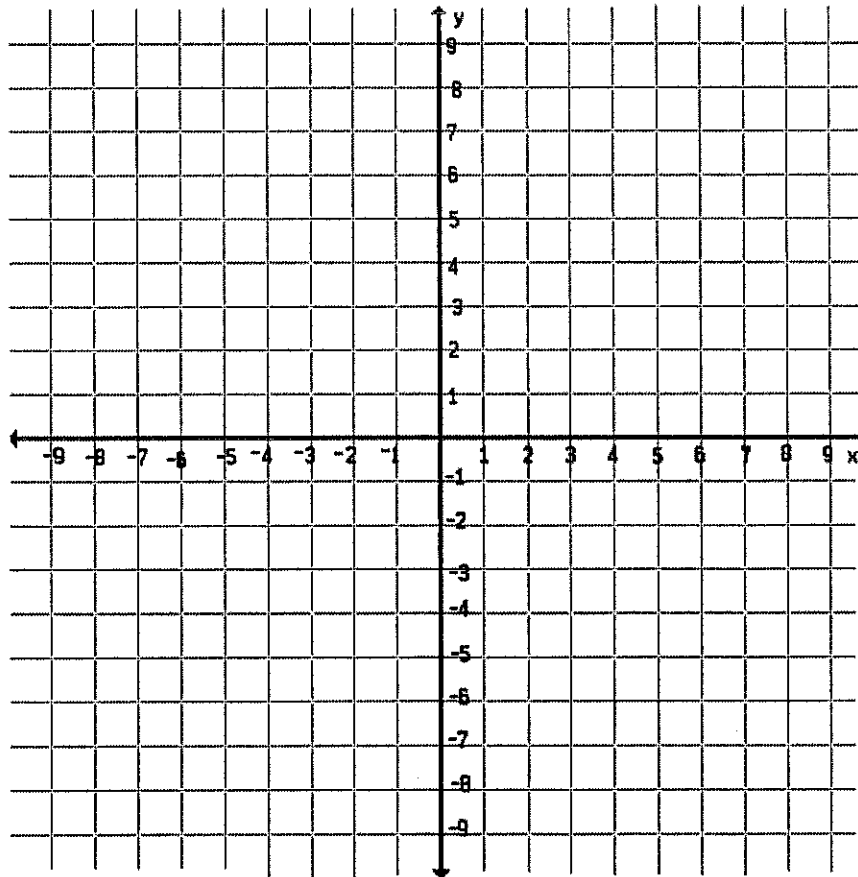
Directions:

- Graph the pre-image.
- Rotate the figure as directed.
- Use different colors to label each translation.
- Show all work.

Example 1: A Triangle NGA

*Note – Assume you are rotating counterclockwise around the origin unless otherwise stated.

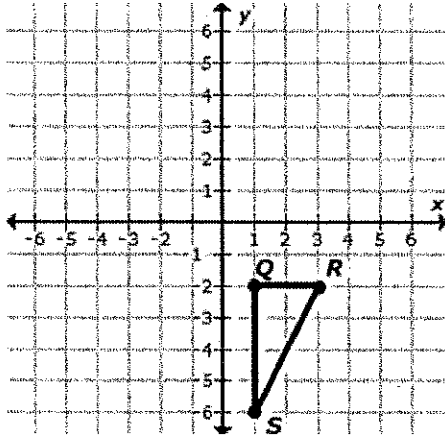
<u>Pre-Image</u>	<u>Rotated CCW 90° (CW 270°)</u>	<u>Rotated CCW 180° (CW 180°)</u>	<u>Rotated CCW 270° (CW 90°)</u>
	(-y, x)	(-x, -y)	(y, -x)
N(2, 0)	(,)	(,)	(,)
G(4, 4)	(,)	(,)	(,)
A(5, -2)	(,)	(,)	(,)



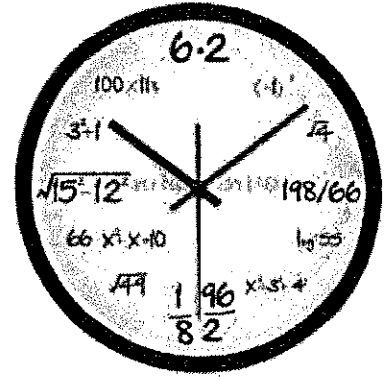
You Try:

Rotate each figure about the origin using the given angle and direction.

1. 90° Counterclockwise



Pre-image	Image



2. Rotate a trapezoid with a pre-image of $(3,2), (7,2), (1,-1), (9,-1)$ clockwise around the origin 180° .

Pre-image	Image



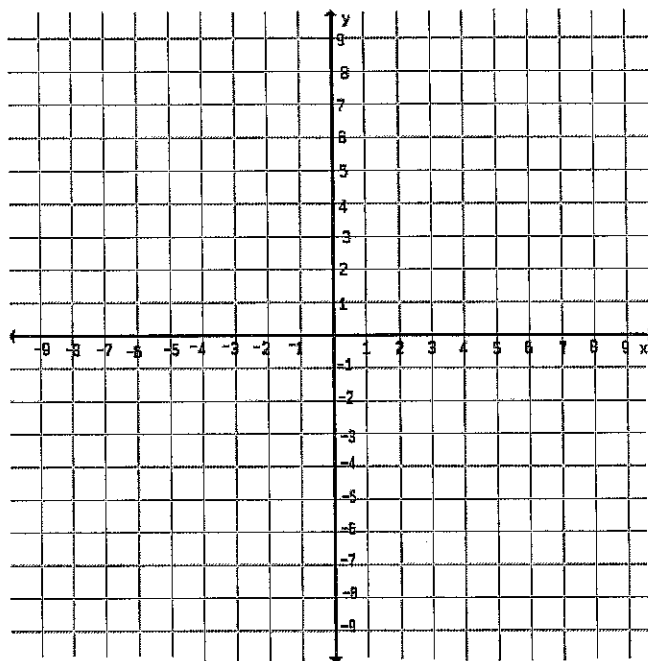
Example 2: A Triangle LAP

Rotate 270° counterclockwise
around the point $(1, 2)$.

$L(7, 3)$

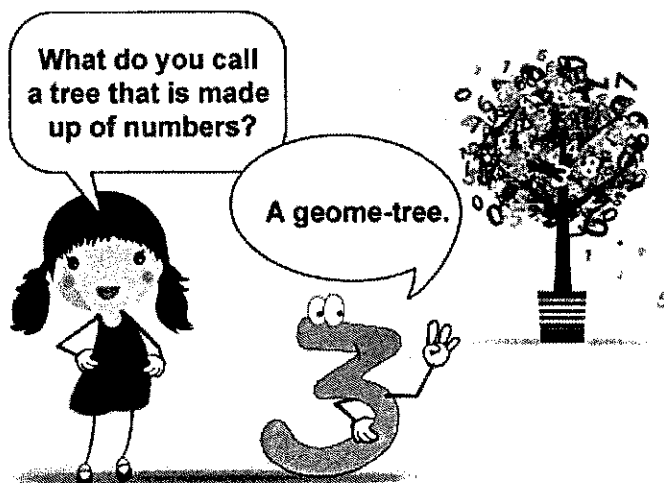
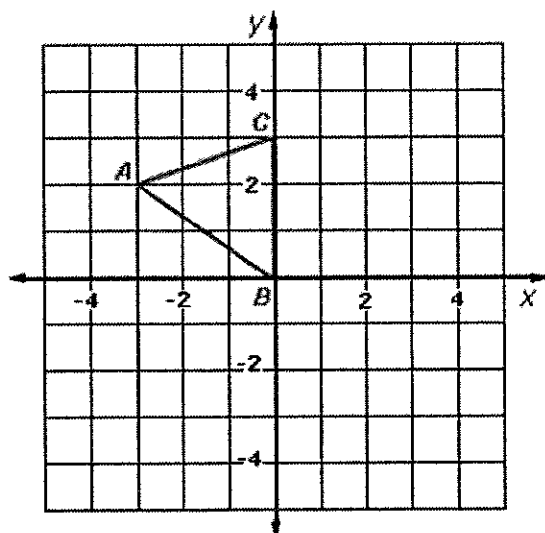
$A(5, 5)$

$P(5, 3)$



You Try:

1. Rotate 270° clockwise around the point $(0, -2)$.

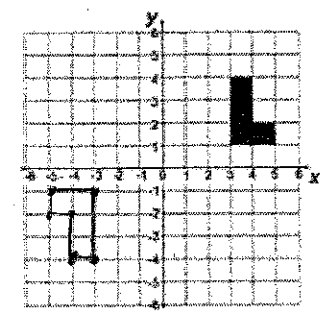


Practice Makes Better

1. Where will the L-Shape be if it is...?

a. rotated 180° around the origin?
the origin?

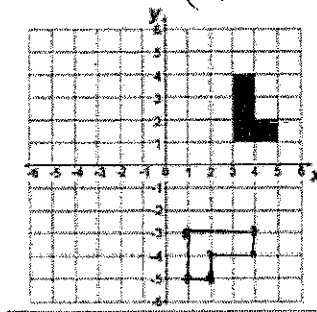
$$(-x, -y)$$



Pre-image	Image
(3, 1)	(-3, -1)
(5, 1)	(-5, -1)
(5, 2)	(-5, -2)
(4, 2)	(-4, -2)
(4, 4)	(-4, -4)
(3, 4)	(-3, -4)

b. rotated 90° clockwise around

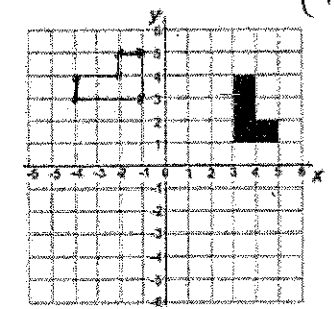
$$(y, -x)$$



Pre-image	Image
(3, 1)	(1, -3)
(5, 1)	(1, -5)
(5, 2)	(2, -5)
(4, 2)	(2, -4)
(4, 4)	(4, -4)
(3, 4)	(4, -3)

c. rotated 90° counterclockwise
around the origin?

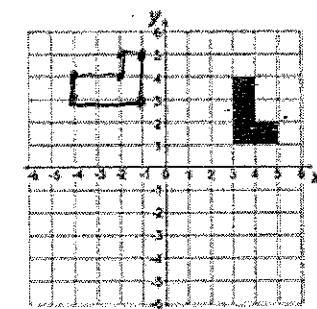
$$(-y, x)$$



Pre-image	Image
(3, 1)	(-1, 3)
(5, 1)	(-1, 5)
(5, 2)	(-2, 5)
(4, 2)	(-2, 4)
(4, 4)	(-4, 4)
(3, 4)	(-4, 3)

d. rotated 270° clockwise around
the origin?

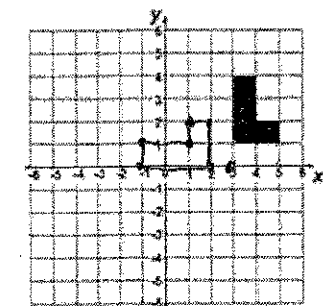
$$(-y, x)$$



Pre-image	Image
(3, 1)	(-1, 3)
(5, 1)	(-1, 5)
(5, 2)	(-2, 5)
(4, 2)	(-2, 4)
(4, 4)	(-4, 4)
(3, 4)	(-4, 3)

e. rotated 90° counterclockwise
around the point (3, 0)?

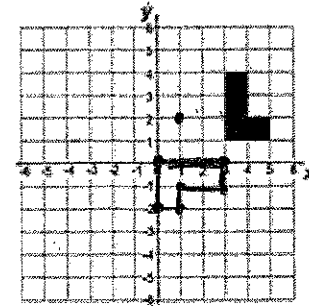
$$(-y, x)$$



Pre-image	Image
(3, 1)	(0, 1)
(5, 1)	(2, 1)
(5, 2)	(2, 2)
(4, 2)	(1, 2)
(4, 4)	(1, 4)
(3, 4)	(0, 4)

f. rotated 90° clockwise around the
point (1, 2)?

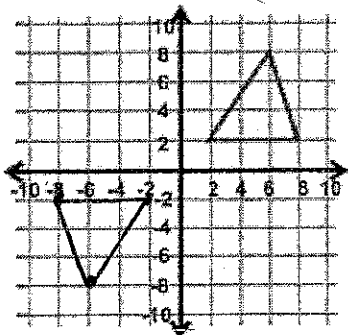
$$(y, -x)$$



Pre-image	Image
(3, 1)	(2, -1)
(5, 1)	(4, -1)
(5, 2)	(4, 0)
(4, 2)	(3, 0)
(4, 4)	(3, 2)
(3, 4)	(2, 2)

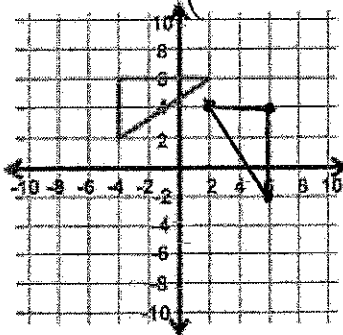
2. Rotate each figure about the origin using the given counterclockwise angle.

a. 180° $(-x, -y)$



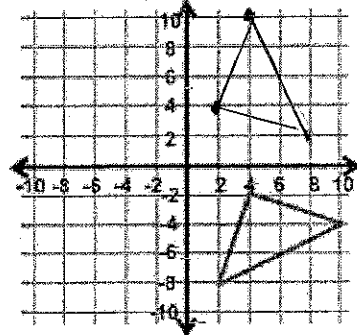
Pre-image	Image
$(2, 2)$	$(-2, -2)$
$(6, 2)$	$(-6, -2)$
$(6, 8)$	$(-6, -8)$

b. 270° $(y, -x)$



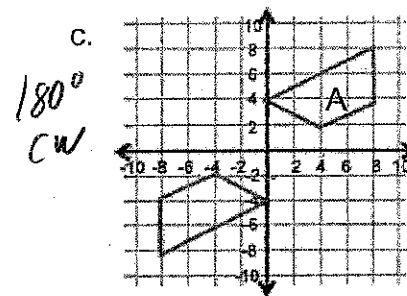
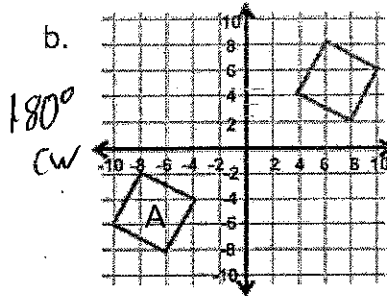
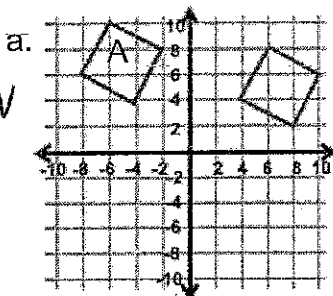
Pre-image	Image
$(-4, 2)$	$(2, 4)$
$(2, 6)$	$(6, -2)$
$(-4, 6)$	$(6, 4)$

c. 90° $(-y, x)$



Pre-image	Image
$(4, -2)$	$(2, 4)$
$(10, -4)$	$(4, 10)$
$(2, -8)$	$(8, 2)$

3. Find the angle of rotation for the graphs below. The center of rotation is the origin, and the figure labeled A is the pre-image. Your answer will be 90° , 270° , or 180° clockwise.



4. Find the coordinates of the vertices of each figure after the given transformation.

a) Rotation 180° about the origin $(-x, -y)$

$Z(-1, -5), K(-1, 0), C(1, 1), N(3, -2)$

$Z'(1, 5)$ $K'(1, 0)$ $C'(-1, -1)$ $N'(-3, 2)$

b) Rotation 180° about the origin $(-x, -y)$

$L(1, 3), Z(5, 5), F(4, 2)$

$L'(-1, -3)$ $Z'(-5, -5)$ $F'(-4, -2)$

c) Rotation 90° clockwise about the origin

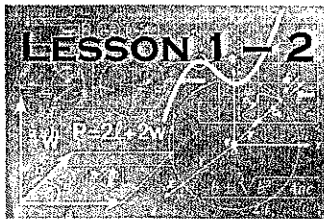
$S(1, -4), W(1, 0), J(3, -4)$ $(y, -x)$

$S'(-4, -1)$ $W'(0, -1)$ $J'(-4, -3)$

d) Rotation 180° about the origin

$V(-5, -3), A(-3, 1), G(0, -3)$ $(-x, -y)$

$V'(5, 3)$ $A'(3, -1)$ $G'(0, 3)$



Reflections and Translations

Directions:

- Graph the original coordinates.
- Graph its reflection over both the x-axis and the y-axis.
- Use different colors to label each reflection.
- Show all work.

Vocabulary:

- Reflection
- Reflection Line
- Translation

Example 1: Reflecting a Triangle

Triangle MNP with vertices M(-8, 4), N(-3, 8), and P(-2, 2)

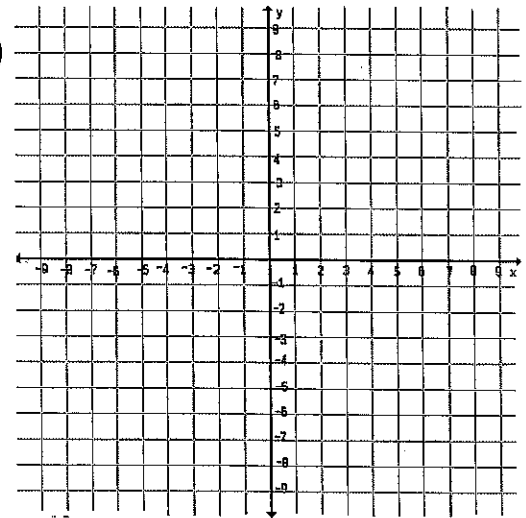
Pre-image

Reflected over x-axis (__, __)

M(-8, 4)

N (-3, 8)

P (-2, 2)



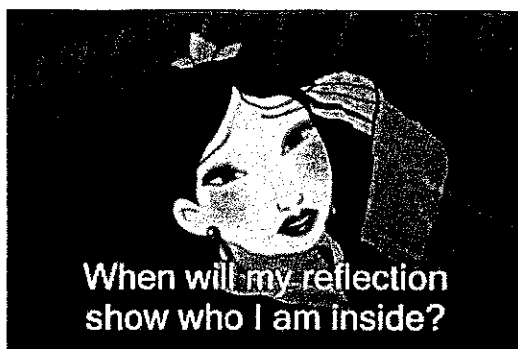
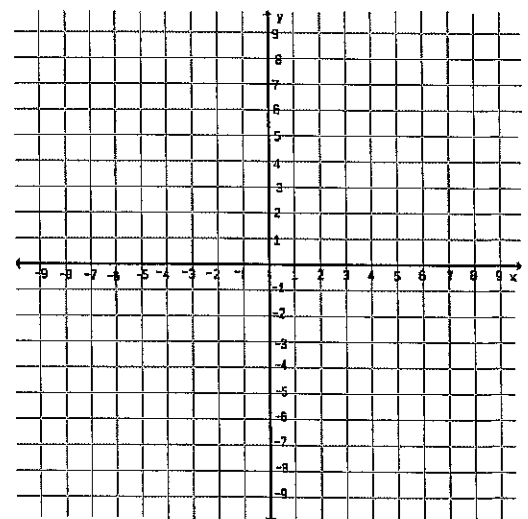
Pre-image

Reflected over y-axis (__, __)

M(-8, 4)

N (-3, 8)

P (-2, 2)



Example 2: Reflecting a Quadrilateral

Quadrilateral WXYZ with vertices $W(-3, 0)$, $X(1, -2)$, $Y(-3, -6)$, and $Z(-7, -2)$

Pre-image

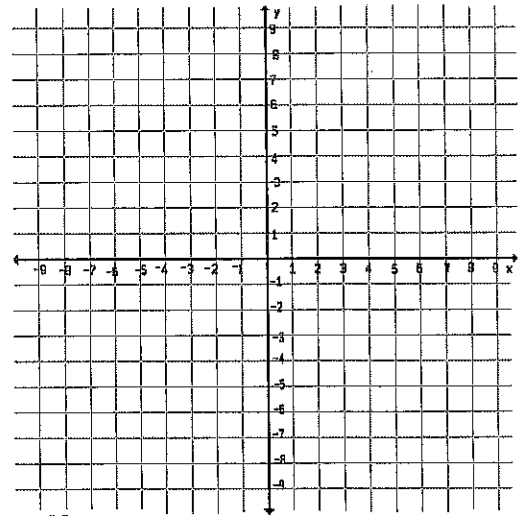
Reflected over $y = 1$

$W(-3, 0)$

$X(1, -2)$

$Y(-3, -6)$

$Z(-7, -2)$



Pre-image

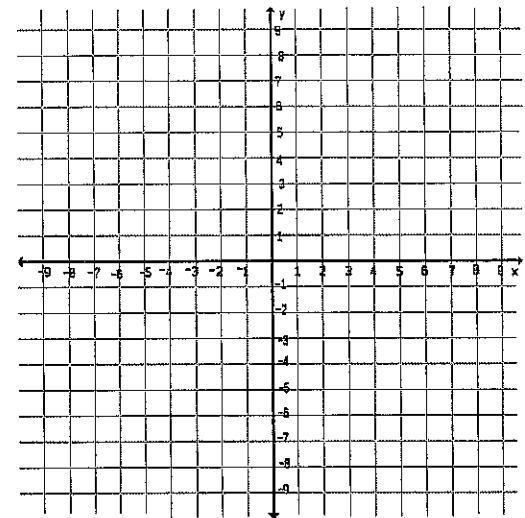
Reflected over $x = -1$

$W(-3, 0)$

$X(1, -2)$

$Y(-3, -6)$

$Z(-7, -2)$



You Try:

1. Pre-image

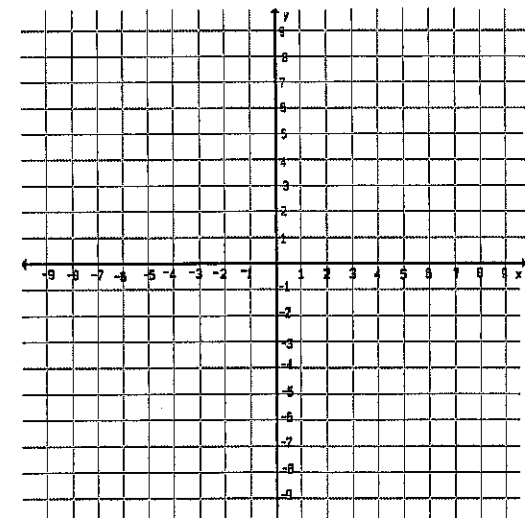
Reflected over $x = 2$

$M(7, 2)$

$A(1, 2)$

$T(1, 5)$

$H(4, 6)$



Example 3: Reflecting a Trapezoid

Trapezoid PQRS with vertices $P(4, -2)$, $Q(8, -2)$, $R(8, -5)$, and $S(2, -5)$

Pre-image

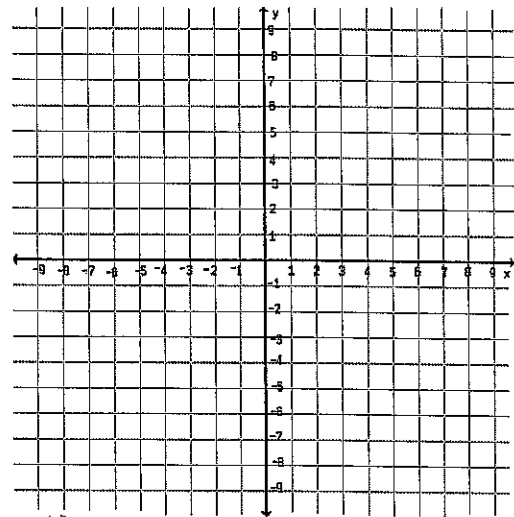
Reflected over $y = x$

$P(4, -2)$

$Q(8, -2)$

$R(8, -5)$

$S(2, -5)$



Pre-image

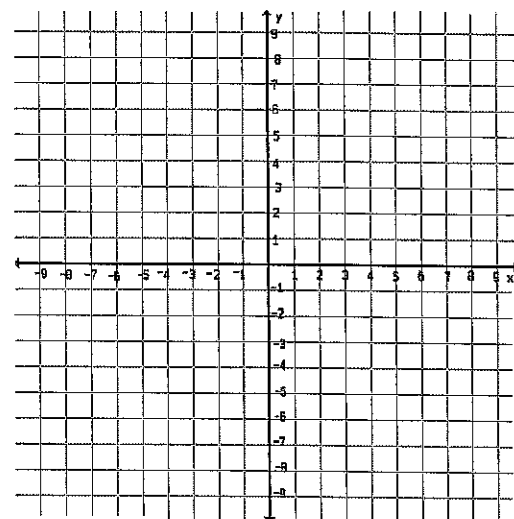
Reflected over $y = -x$

$P(4, -2)$

$Q(8, -2)$

$R(8, -5)$

$S(2, -5)$



You Try:

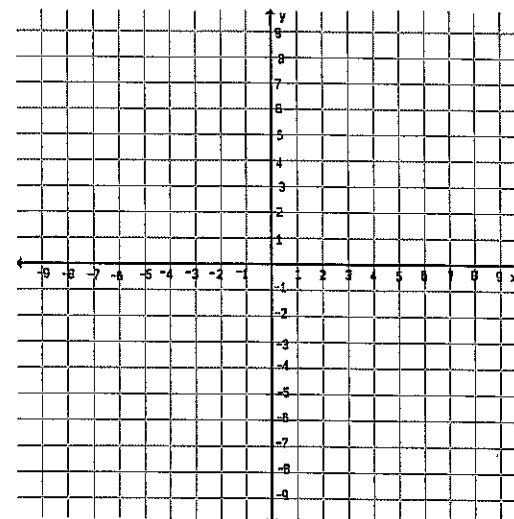
1. Preimage

Reflected over $y = x$

$A(-3, 1)$

$B(-5, 5)$

$C(-1, 2)$



Example 4: Translating a Triangle

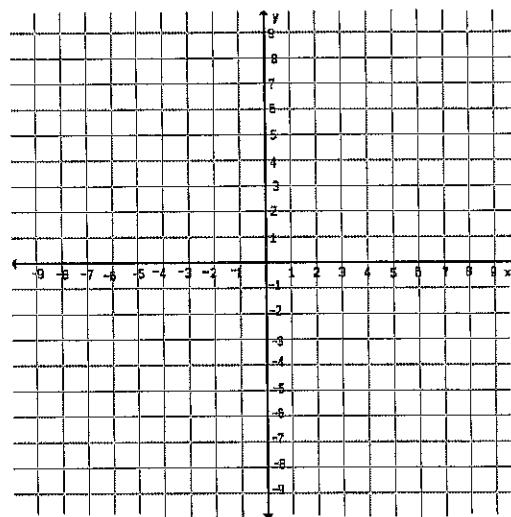
Triangle EFG with vertices E(-5, -2) , F (-2, 3) , and G (2, -3)
translated $(x, y) \rightarrow (x+6, y + 3)$

Preimage Translate $(x, y) \rightarrow (x + 6, y + 3)$

E(-5, -2)

F (-2, 3)

G (2, -3)



You Try:

Square SQAR with vertices S(2, 1) , Q(4, 3) , A(2, 5) and R(0, 3)
Translate (x, y) left 1 and up 3

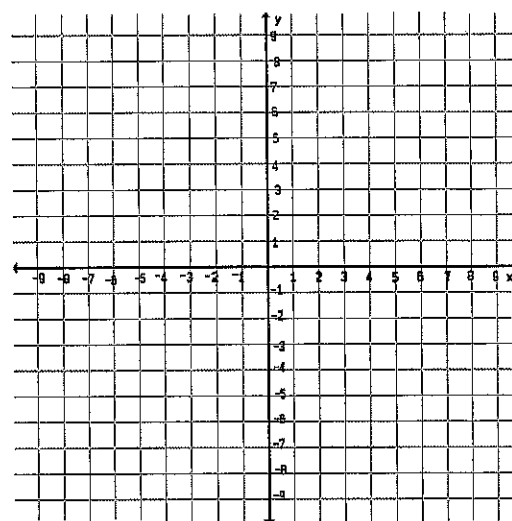
Preimage Translate (x, y) left 1 and up 3

S(2, 1)

Q(4, 3)

A(2, 5)

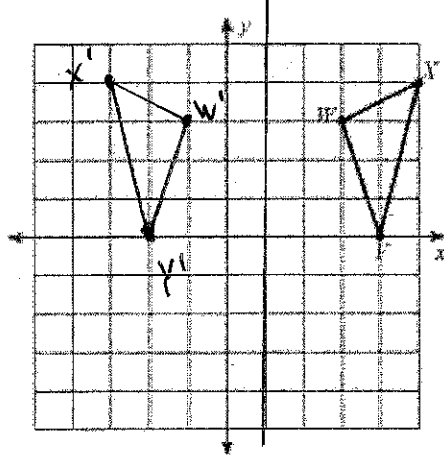
R(0, 3)



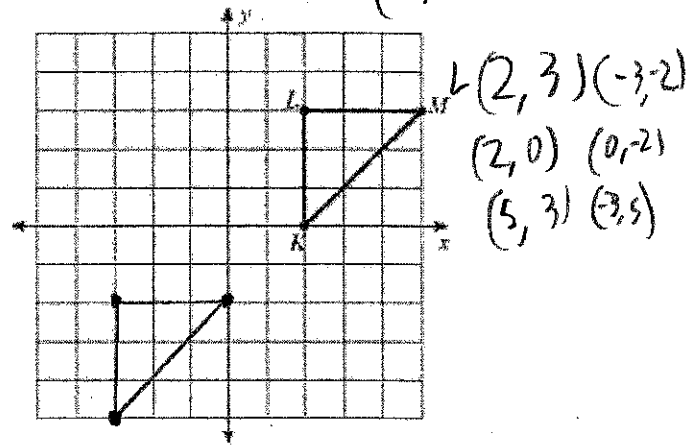
Practice Makes Better

Graph the image of the figure using the transformation given.

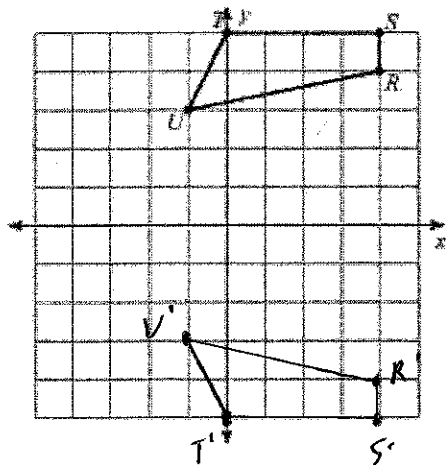
1) reflection across $x = 1$



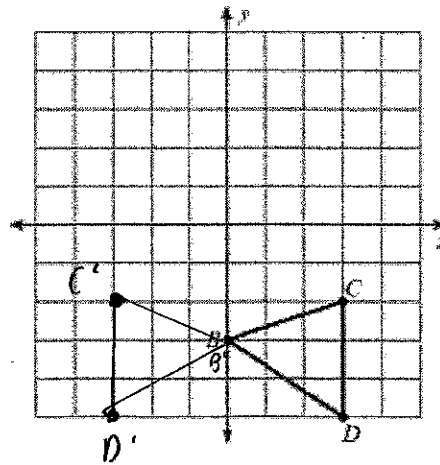
2) reflection across $y = -x$ $(-y, -x)$



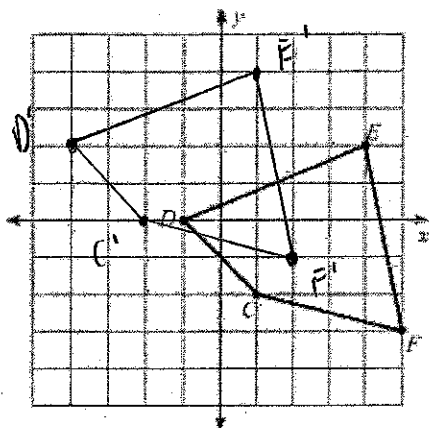
3) reflection across the x-axis



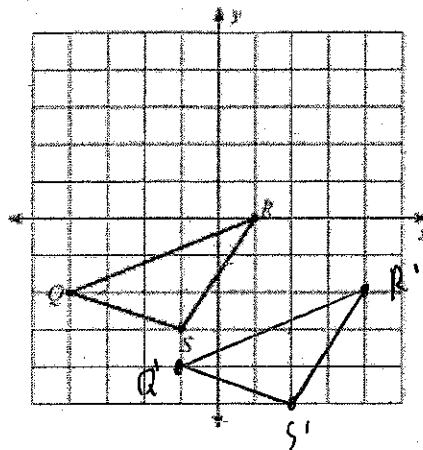
4) reflection across the y-axis



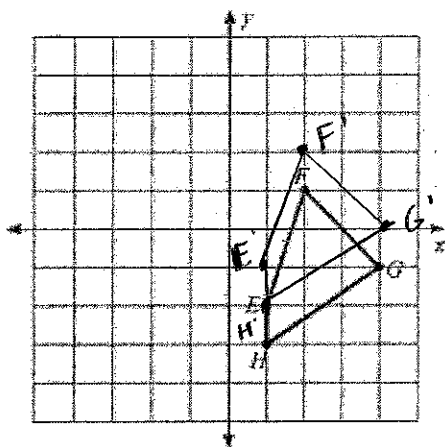
5) translation: 3 units left and 2 units up



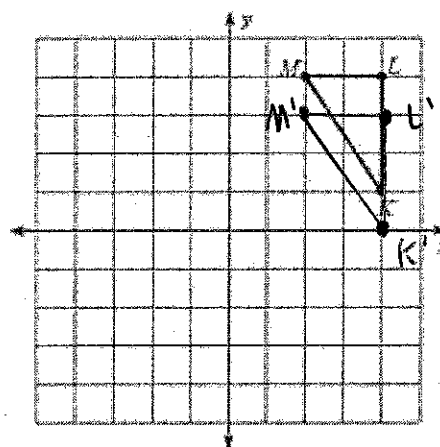
6) translation: 3 units right and 2 units down



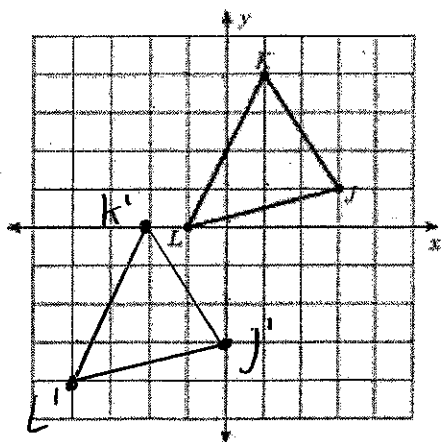
7) translation: 1 unit up



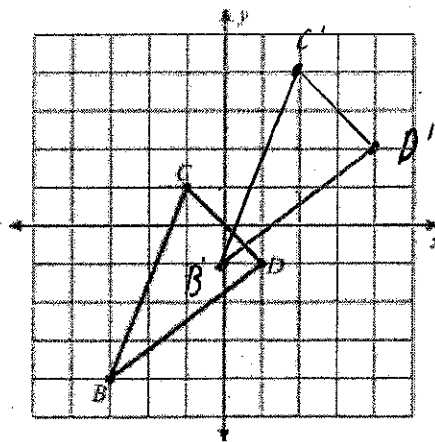
8) translation: $(x, y) \rightarrow (x, y - 1)$



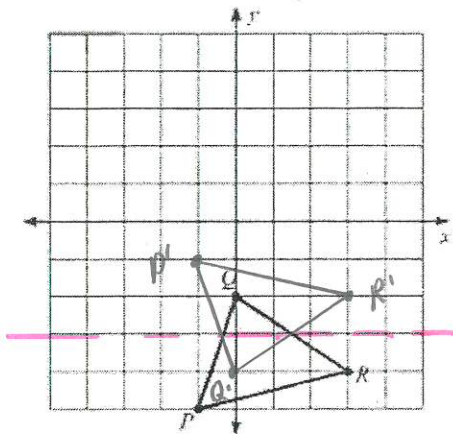
9) translation: $(x, y) \rightarrow (x - 3, y - 4)$



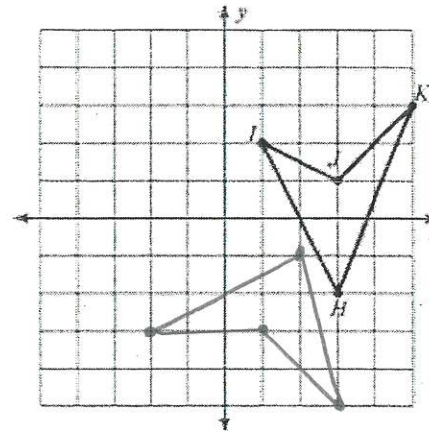
10) translation: $(x, y) \rightarrow (x + 3, y + 3)$



11) reflection across $y = -3$



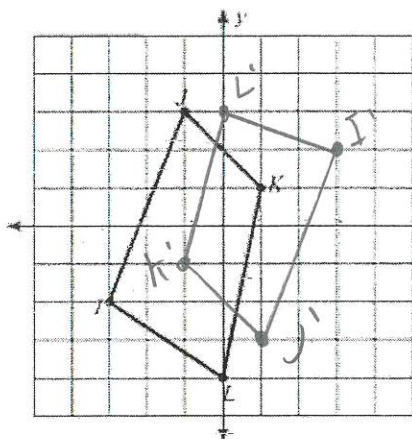
12) rotation 90° clockwise about the origin



$$(y, -x)$$

$$\begin{array}{ll} H(3, 2) & (2, -3) \\ I(1, 2) & (2, -1) \\ J(3, 1) & (1, -3) \\ K(5, 3) & (3, -5) \end{array}$$

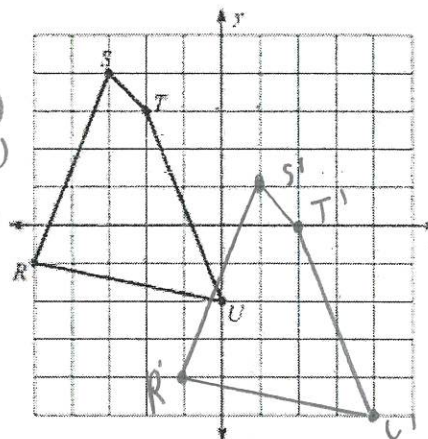
13) rotation 180° about the origin



$$(-x, -y)$$

$$\begin{array}{ll} J(-1, 3) & (1, -3) \\ K(1, 1) & (-1, -1) \\ L(0, -3) & (0, 3) \\ I(-3, -2) & (3, 2) \end{array}$$

14) translation: $(x, y) \rightarrow (x + 4, y - 3)$



Find the coordinates of the vertices of each figure after the given transformation.

15) translation: 3 units left and 3 units down
Z(1, 0), Y(1, 4), X(3, 2)

$$Z'(-2, -3) \quad Y'(-2, 1) \quad X'(0, -1)$$

16) translation: 1 unit left and 2 units down
J(0, -3), K(0, -2), L(3, -2), M(4, -3)

$$J'(-1, -5) \quad K'(-1, -4) \quad L'(2, -4) \quad M'(3, -5)$$

17) reflection across the x-axis
W(-4, -4), X(-4, -2), Y(-2, -1)

$$W'(-4, 4) \quad X'(-4, 2) \quad Y'(-2, 1)$$

18) reflection across the y-axis
W(0, 3), X(2, 5), Y(3, 4)

$$W'(0, 3) \quad X'(-2, 5) \quad Y'(-3, 4)$$

19) rotation 180° about the origin $(-x, -y)$
 $H(-3, 0), I(-5, 3), J(-5, 4), K(-1, 1)$

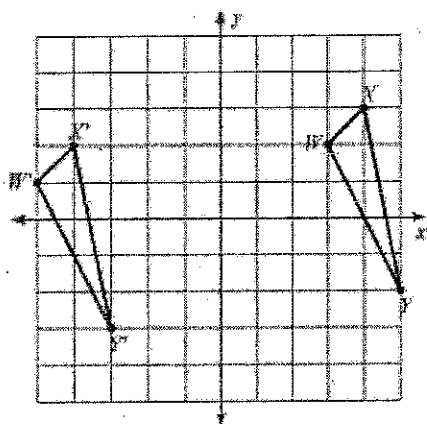
$H'(3, 0) I'(5, -3) J'(5, -4) K'(1, -1)$

20) rotation 90° clockwise about the origin $(y, -x)$
 $N(-1, -1), M(-3, 2), L(0, 5), K(2, 0)$

$N'(-1, 1) M'(2, 3) L'(5, 0) K'(0, -2)$

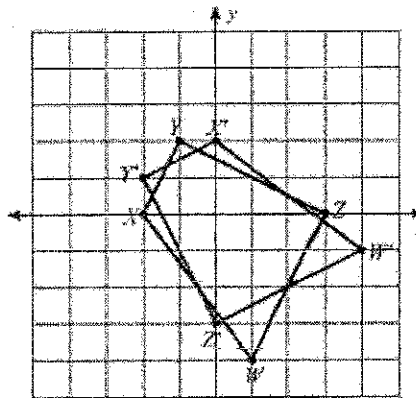
Write a rule to describe each transformation.

21)



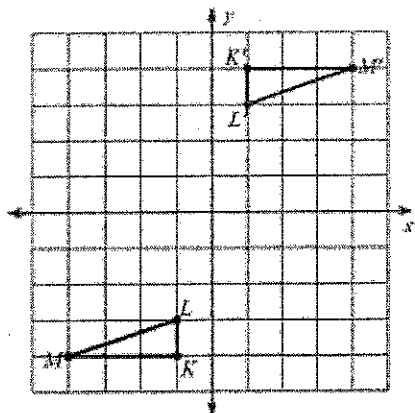
$(x+7, y+1)$

22)



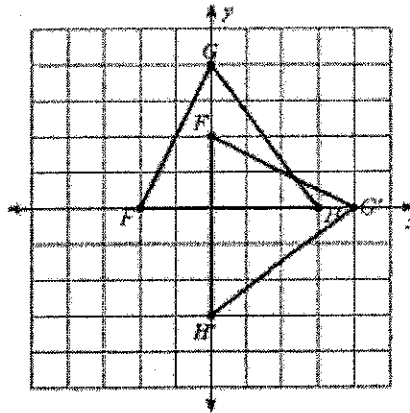
Reflected over
 $y = -x$

23)

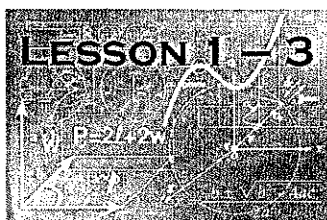


Rotate 180°

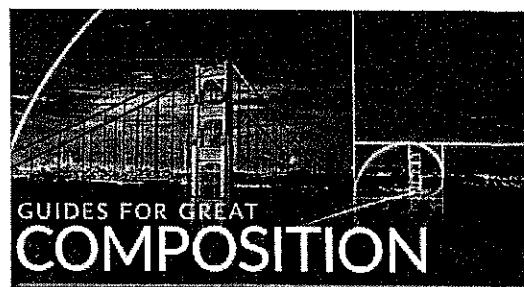
24)



Rotate 90° CW
 Rotate 270° CCW



Compositions of Transformations



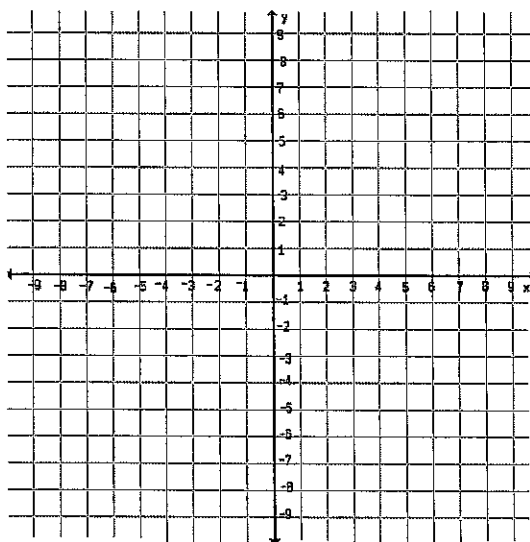
A _____, also known as composition of transformations is a series of multiple transformations performed one after the other.

Directions:

- Graph the original coordinates.
- Then, apply the listed transformations.
- Graph the new images.
- Be sure to draw each new image in a new color.

Example 1:

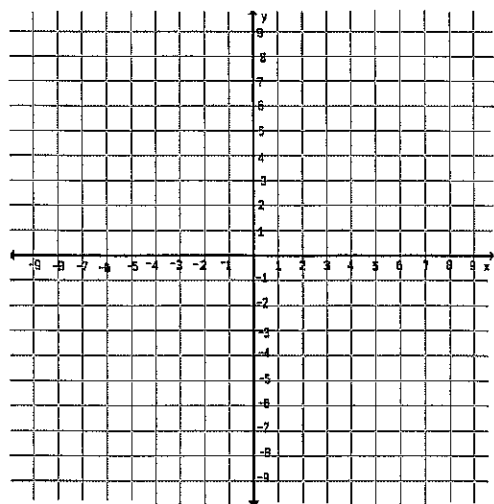
Triangle KLM has vertices K(4, -1), L(5, -2), and M(1, -4). Rotate Triangle KLM 180° about the origin and then reflect it across the y-axis.



You Try:

Trapezoid MATH has vertices as M(-4, 0), A(0,2), T(0,-2), H(-4,-2).

Translate Trapezoid MATH from $(x, y) \rightarrow (x - 2, y + 3)$ and then reflect it over the y-axis.



***Note:** A glide reflection is a type of opposite isometry. It consists of a reflection and a translation along the line reflection.

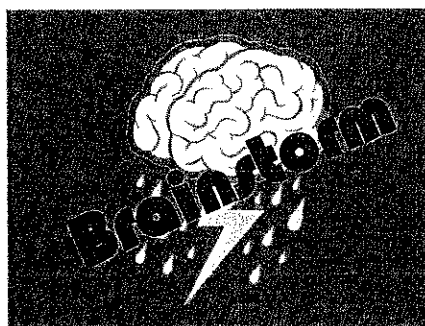
For some compositions, the order in which you complete the transformations is important. However, there are some compositions which are commutative (order does not matter).

Go back to Example 1 and the You Try. Are these compositions commutative? Or do you have to stick to the order given?

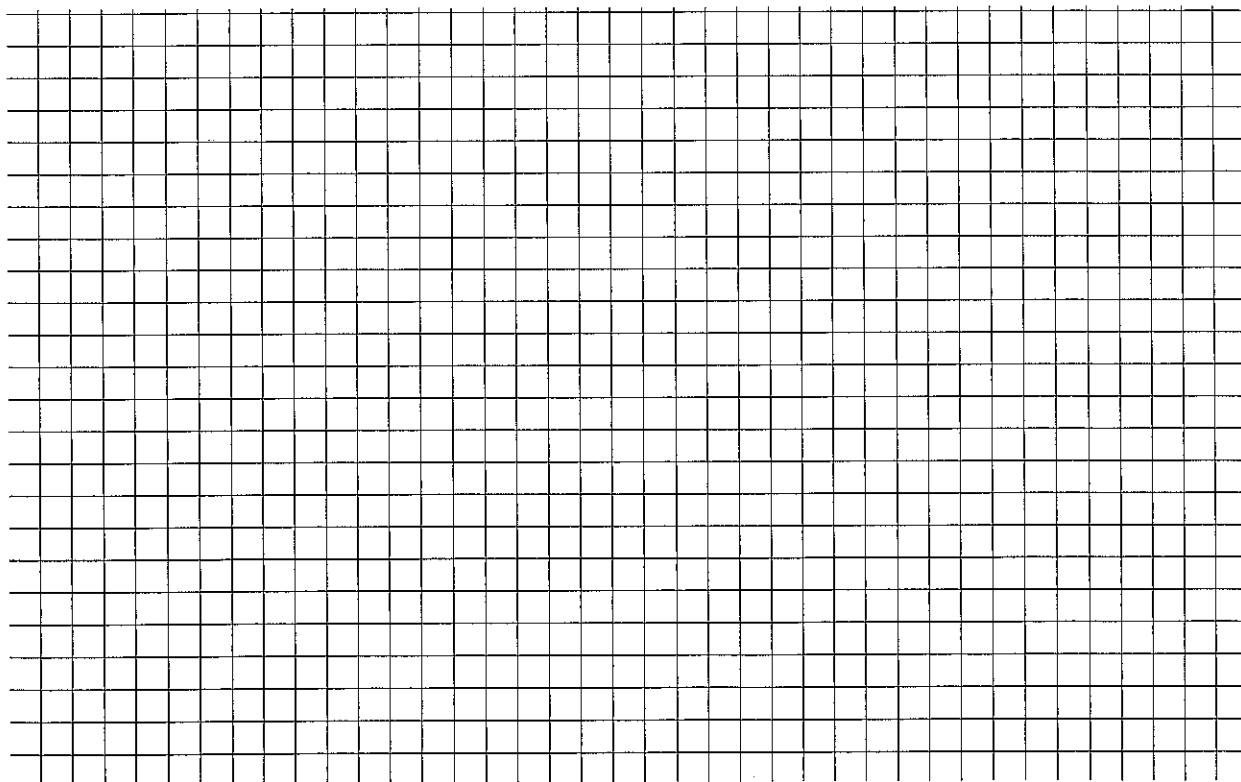
Example 1: _____ You Try: _____

You Try: What pairings of transformations can you think of for which order would matter? For which compositions would order not matter? Use the grid below to experiment as needed.

Order Matters



Commutative

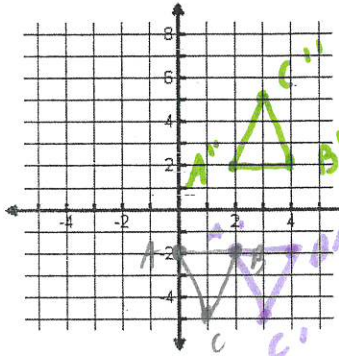


Practice Makes Better

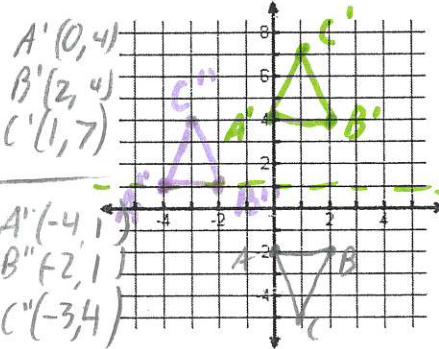
Graph the image of $A(0, -2)$, $B(2, -2)$ and $C(1, -5)$ & each transformation.

1. Translation: $(x + 2, y)$,
then Reflection: across
the x -axis

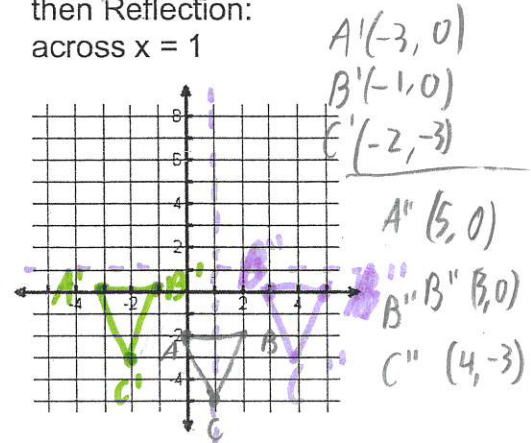
$$\begin{array}{l} A'(2, -2) \\ B'(4, -2) \\ C'(3, -5) \\ \hline A''(2, 2) \\ B''(4, 2) \\ C''(3, 5) \end{array}$$



2. Reflection: across $y = 1$,
then Translation:
 $(x - 4, y - 3)$

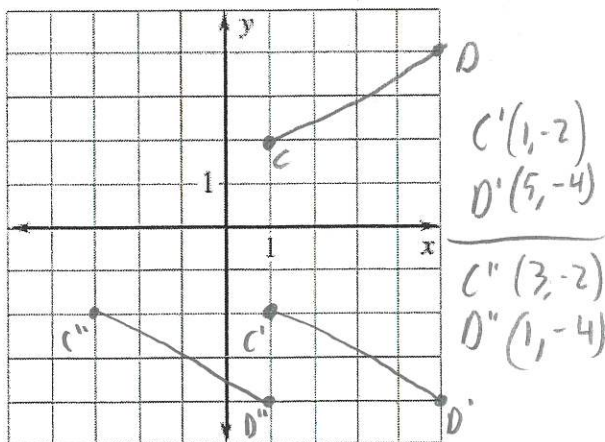


3. Translation: $(x - 3, y + 2)$,
then Reflection:
across $x = 1$

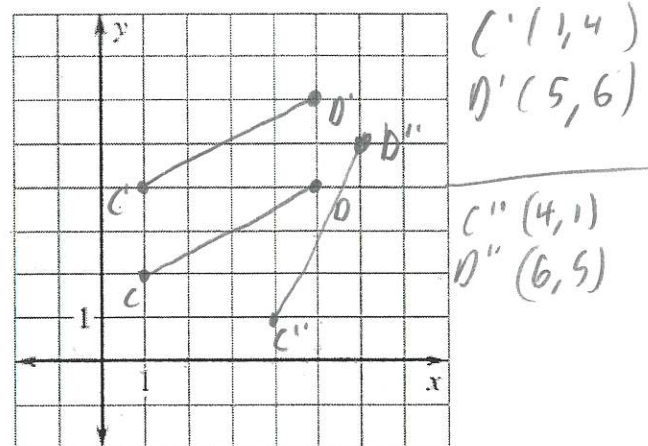


The endpoints of CD are $C(1, 2)$ and $D(5, 4)$. Graph the pre-image of CD & each transformation.

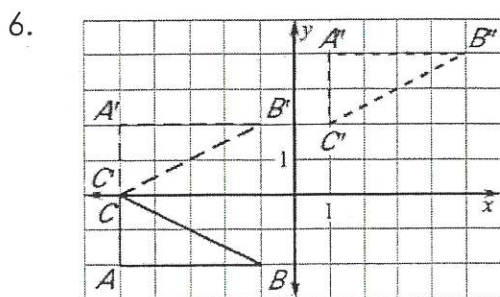
4. Reflection: across the x -axis,
followed by
Translation: $(x - 4, y)$



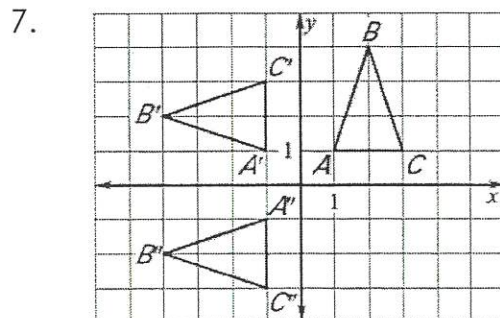
5. Translation: $(x, y + 2)$, followed by
Reflection: across $y = x$
 (y, x)



Specify the sequence of transformations that occurred.



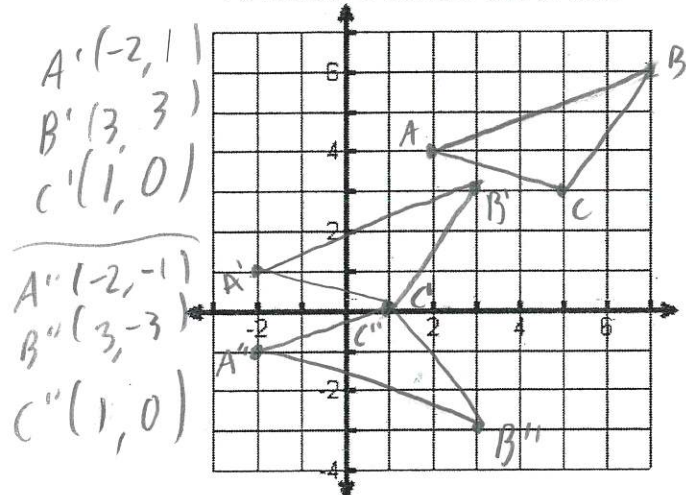
Reflected over x -axis
Translated $(x + 7, y + 2)$



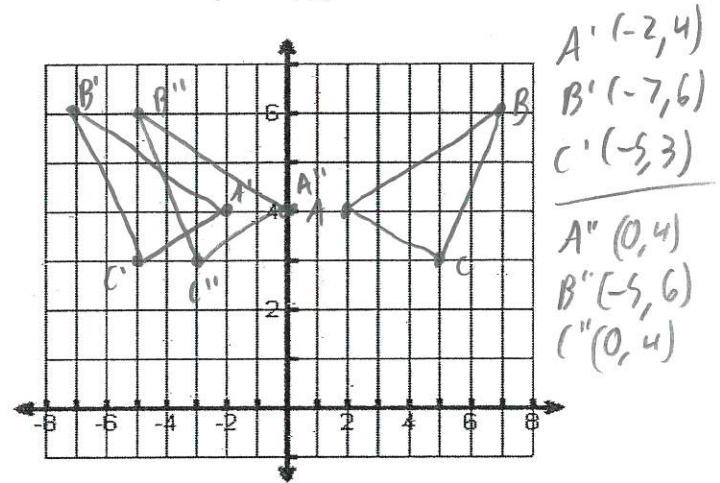
Rotate
 90° (CW,
then reflect
over x -axis

The vertices of $\triangle ABC$ are $A(2,4)$, $B(7,6)$, and $C(5,3)$. Graph the pre-image of $\triangle ABC$ & each transformation.

8. Translation: $(x - 4, y - 3)$, followed by
Reflection: across the x-axis

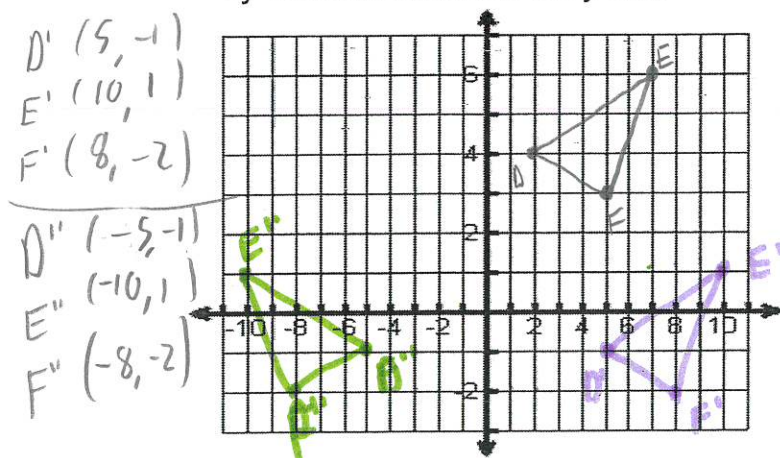


9. Reflection: across the y-axis,
followed by
Translation: $(x + 2, y)$

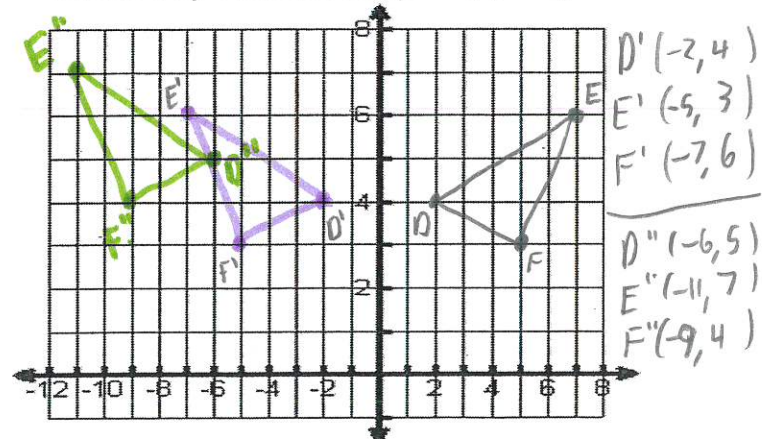


The vertices of $\triangle DEF$ are $D(2,4)$, $E(7,6)$, and $F(5,3)$. Graph the pre-image of $\triangle DEF$ & each transformation.

10. Translation: $(x + 3, y - 5)$, followed by
Reflection: across the y-axis

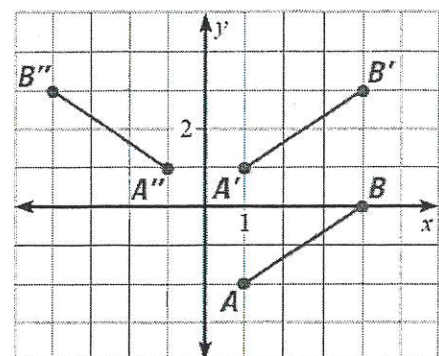


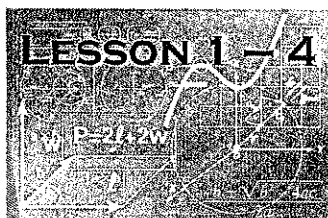
11. Reflection: across the y-axis,
followed by Translation: $(x - 4, y + 1)$



In the diagram, AB is the pre-image of a combination.

12. Which segment is a translation of AB?
 $A'B'$
 13. Write a rule to describe the translation for #12.
 $(x, y+3)$
 14. Which segment is a reflection of $A'B'$?
 $A''B''$
 15. Name the line of reflection for #14.
 $y\text{-axis } (x=0)$



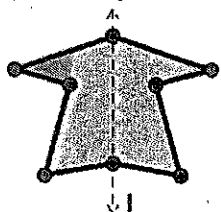


Mapping a Figure onto Itself

Vocabulary:

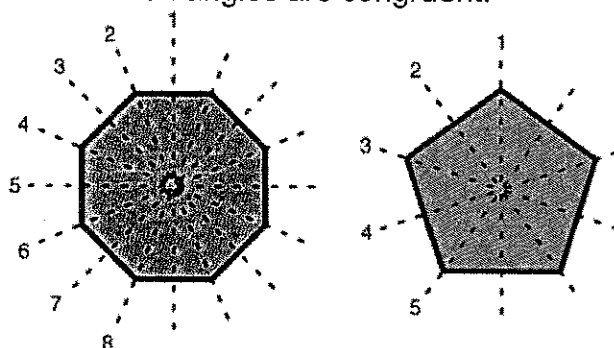
- Line Symmetry
- Reflection Line
- Rotational Symmetry

The ability to carry, or map, an image onto itself shows that the figure has symmetry. We are going to discuss two types of symmetry that a shape can have; line symmetry and rotational symmetry.

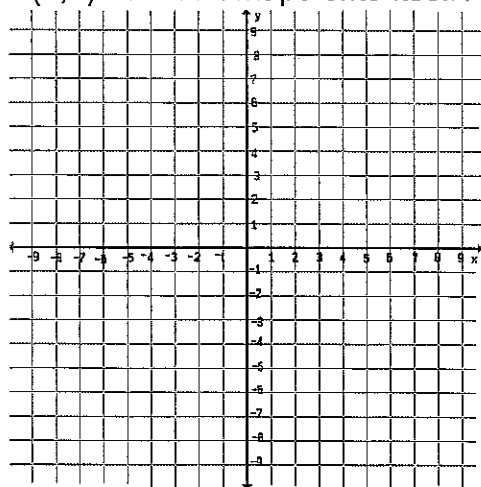


A figure in a plane has a line of symmetry if the figure can be mapped onto itself by a _____.

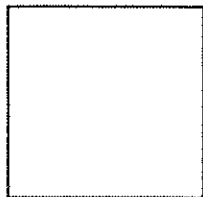
The maximum lines of symmetry that a polygon can have are equal to its number of sides. The maximum is always found in a regular polygon, because all sides and all angles are congruent.



Example 1: What line could you reflect Triangle ABC with vertices $A(-1,3)$, $B(3,6)$, and $C(7,3)$ so that it maps onto itself? Draw the line on the graph.



You Try: For each figure, identify if it has a line of symmetry or not. If it does, determine how many.



Rotational Symmetry:

A figure has rotational symmetry if there is a center point about which the figure is rotated a number of degrees between 0° and 360° (exclusive) such that the image is congruent to the pre-image.

*We exclude the angles of 0° and 360° because nothing happens in those cases.

The point around which you rotate is called the _____, and the smallest angle you need to turn is called the _____.

This figure has rotation symmetry of 72° , and the center of rotation is the center of the figure:

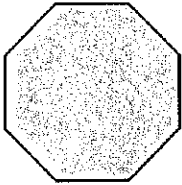
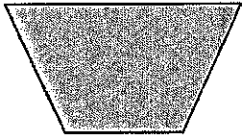
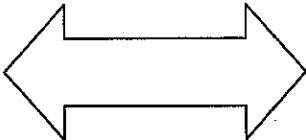



For a regular polygon of n sides, the polygon can be mapped back onto itself by a rotation of x° where $x = \frac{360}{n}$.

Order of Rotational Symmetry: The **order of symmetry** is the number of times the figure coincides with itself as it rotates through 360° .

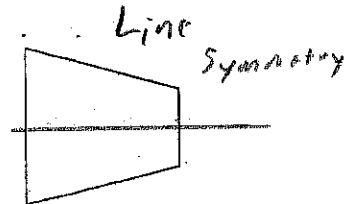
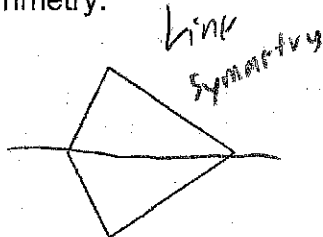
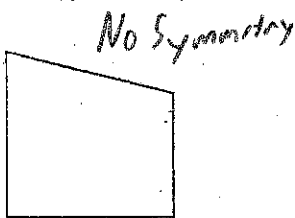
Example 2: What rotation would map a regular pentagon onto itself? What would the order of symmetry be for a regular pentagon?

You Try:

			
Rotational symmetry?	Rotational symmetry?	Rotational symmetry?	Rotational symmetry?
Degrees of Rotation?	Degrees of Rotation?	Degrees of Rotation?	Degrees of Rotation?
Order?	Order?	Order?	Order?

Practice Makes Better

1. State whether the following figures have line symmetry, rotational symmetry, or no symmetry.



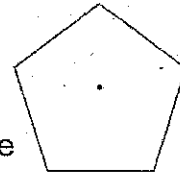
2. A regular pentagon is shown in the diagram. If the pentagon is rotated clockwise about its center, what is the minimum number of degrees needed to carry it onto itself?

A. 54°

B. 72°

C. 108°

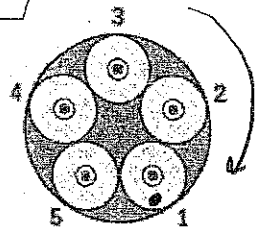
D. 360°



3. Your CD player can hold five compact discs on a rotating tray like one shown.

a. Does the tray have rotational symmetry? Explain:

yes, 72°

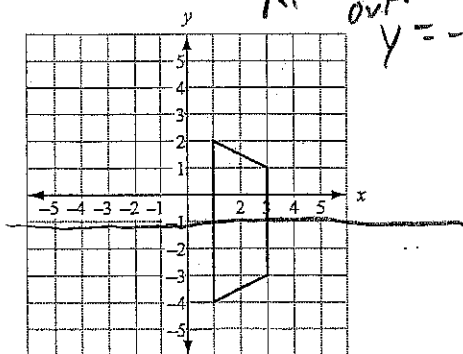


b. The tray can move only clockwise. A CD in position 1 is currently playing. How many degrees must the tray rotate to play a CD in position 3?

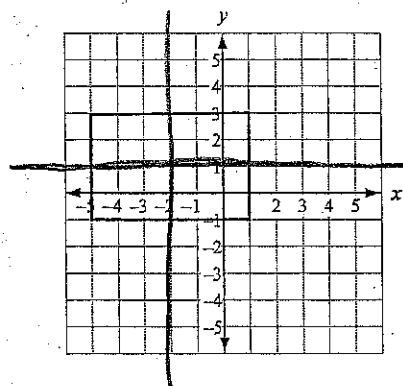
144°

4. Describe every transformation that maps the given figure to itself.

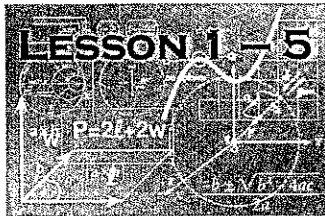
a)



b)



Rotate 180° ,
Reflect over
 $x = -2$,
Reflect over
 $y = 1$



Dilations

A dilation is a transformation that moves each point on the original figure along a straight line drawn from a fixed point, called the center (or point) of dilation. The description of a dilation must include the scale factor (or ratio) and the center of dilation. **Dilation produce figures that are the same shape as the original, but not the same size.**

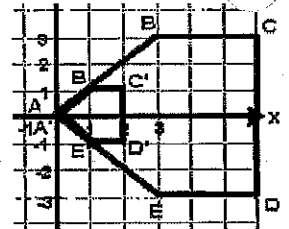
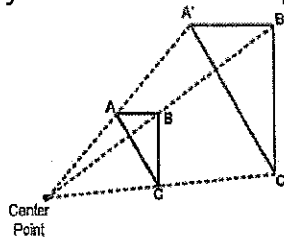
When the scale factor is greater than one, the dilation is called an _____.

When the scale factor is less than one, the dilation is called a _____.

Properties of Dilations

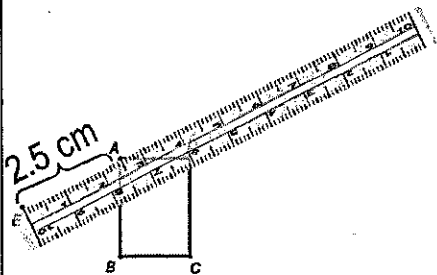
When a figure is dilated:

- The image and the pre-image are the same shape, but not the same size
- Angles are moved to angles of the same measure
- The ratios of corresponding line segments give the scale factor (new/old or image/preimage)
- The area increases by the scale factor squared
- A dilation which does not pass through the center of dilation travels parallel to the original image



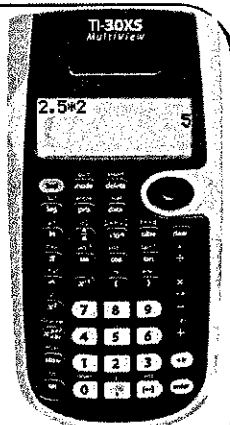
Example 1: Dilate the □ABCD by a factor of 2.0 from point E.

Step 1: Measure the distance from the point of dilation to a point to be dilated (preferably using centimeters).

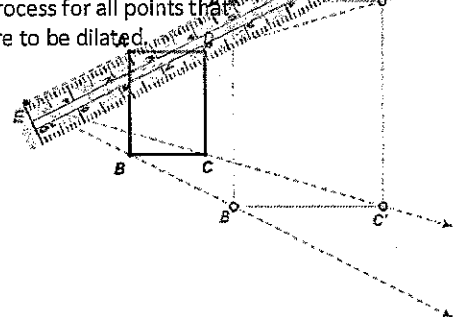


Step 2: Multiply the measured distance by the scale factor.

$$2.5\text{ cm} \times 2.0 = 5\text{ cm}$$

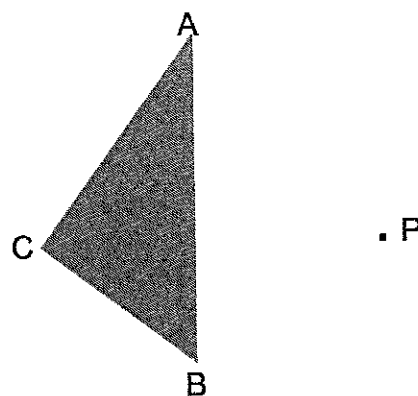


Step 3: With the ruler in the same place as it was in step #1, mark a point at the measured distance determined in step #2 as the image of the original point. Repeat the process for all points that are to be dilated.

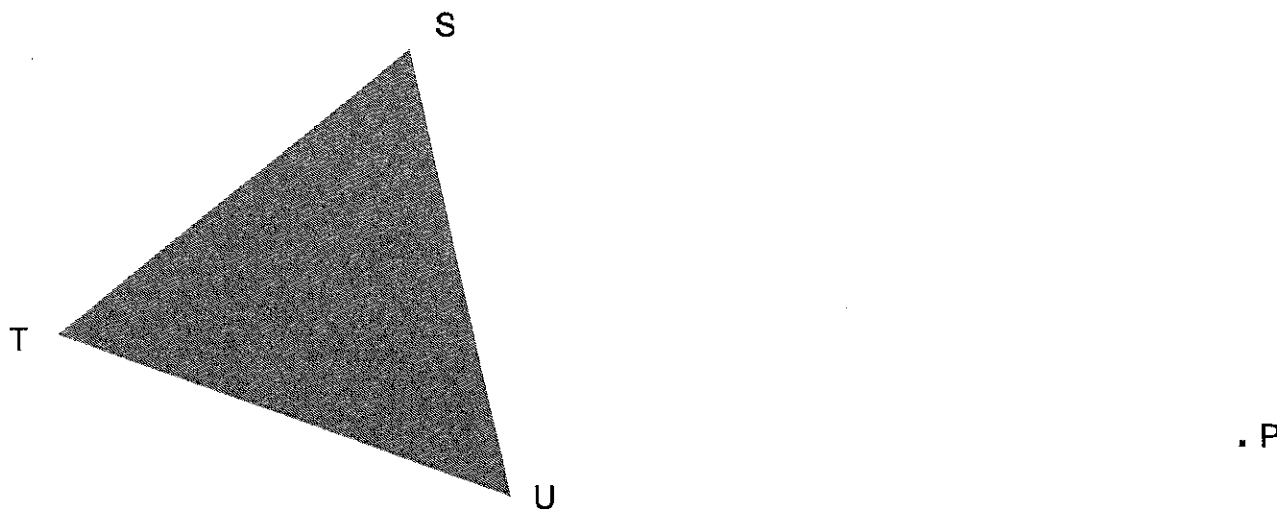


Example 2: Enlarge triangle ABC with P as the center of dilation and scale factors of 2 and 3.

1. Draw and extend the lines from P through each vertex
2. Enlarge $\triangle ABC$ by a factor of 2 (name $\triangle A'B'C'$)
3. Enlarge $\triangle ABC$ by a factor of 3 (name $\triangle A''B''C''$)



You Try: Reduce triangle STU with P as the center of dilation and scale factors of $\frac{1}{2}$ and $\frac{1}{4}$.



4. Draw lines from each vertex to point P.
5. Reduce triangle STU by a factor of $\frac{1}{2}$ (name $\triangle S'T'U'$)
6. Reduce triangle STU by a factor of $\frac{1}{4}$ (name $\triangle S''T''U''$)

Center of Dilation at the Origin

When the center of dilation is the origin, you can just multiply the coordinates of the vertices of the original figure by the scale factor to find the coordinates of the vertices of the dilated figure. That is, the coordinate rule for a dilation with the center dilation at the origin is $(x, y) \rightarrow (kx, ky)$ where k is the scale factor.

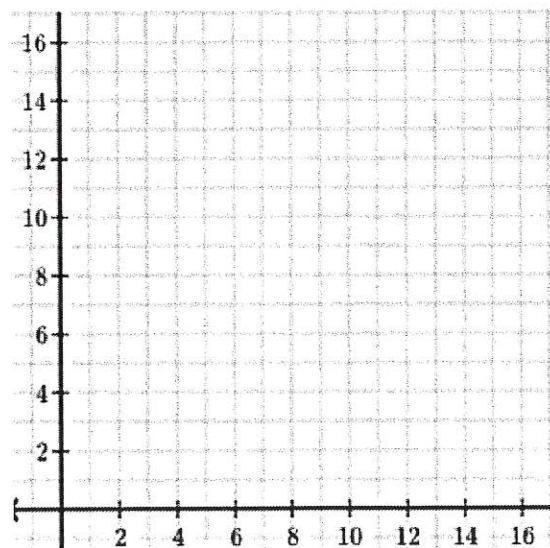
Example 2: Dilate Parallelogram STUV using the origin as the center of dilation and a scale factor $\frac{1}{2}$.

S(8, 10) \rightarrow

T(6, 6) \rightarrow

U (12, 6) \rightarrow

V(14, 10) \rightarrow



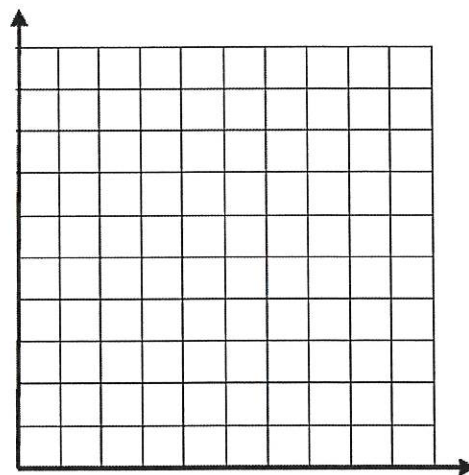
You Try:

1. Dilate $\triangle ABC$ by a factor of 2 with $(0,0)$ as the point of dilation.

A (3, 5) \rightarrow

B(3, 3) \rightarrow

C (5, 3) \rightarrow



Did
You
Know?

Lewis Carrol, the author of Alice in Wonderland was a mathematician who worked primarily in Geometry and Logic. Much of the absurdity in his novels is actually math in disguise!

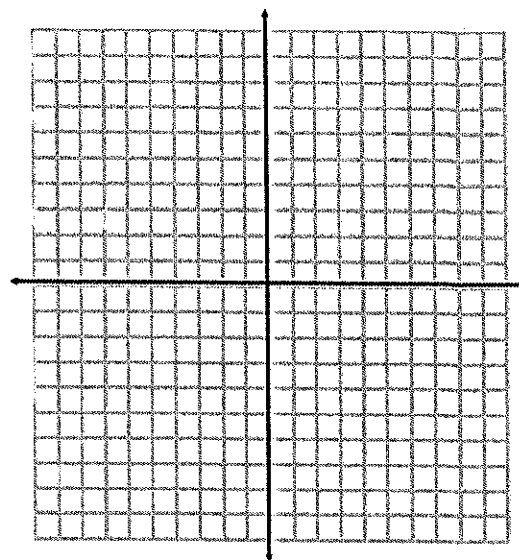


2. $\triangle DEF$ with center of dilation at origin and scale factor of 2.

D (-2, 3) \rightarrow

E (-3, -3) \rightarrow

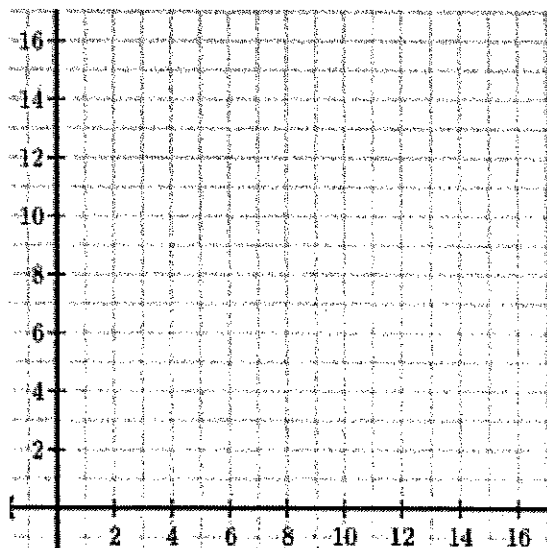
F (4, -3) \rightarrow



Dilate each figure using the given point as the center of dilations.

Unfortunately, there is not easy coordinate rule for making dilations when the center of dilation is a point other than the origin.

Example 3: $\triangle JKL$ with center P (3, 1) and scale factor of 2.

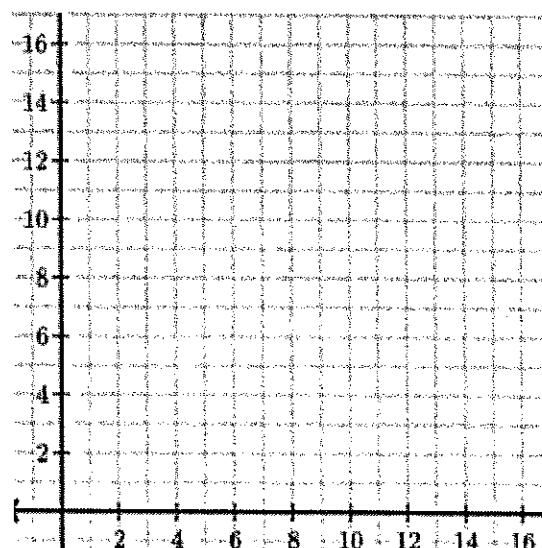


J (3, 7) \rightarrow

K(4, 4) \rightarrow

L (8, 4) \rightarrow

You Try: Trapezoid TPZD with center P(5,5) and scale factor of $1/3$.



T(2,14) \rightarrow

P (5,5) \rightarrow

Z (11,5) \rightarrow

D(14, 14) \rightarrow

Practice Makes Better

For each problem, look at the mapping rule and state whether or not it represents a dilation. If is a dilation, state whether or not the image will be similar to the pre-image.

1. $(x, y) \rightarrow (3x, 1/2y)$

~~Dilation~~ or ~~not~~
Not

If dilation will the image be:

Similar or ~~not~~

2. $(x, y) \rightarrow (x + 6, 6y)$

Dilation or ~~not~~

If dilation will the image be:

Similar or not

3. $(x, y) \rightarrow (2x + 1, 2y + 1)$

Dilation or ~~not~~

If dilation will the image be:

Similar or not

4. $(x, y) \rightarrow (.75x, .75y)$

Dilation or not

If dilation will the image be:

Similar or not

5. $(x, y) \rightarrow (x - 5, y - 5)$

Dilation or not

If dilation will the image be:

Similar or not

6. $(x, y) \rightarrow (4x, 4y)$

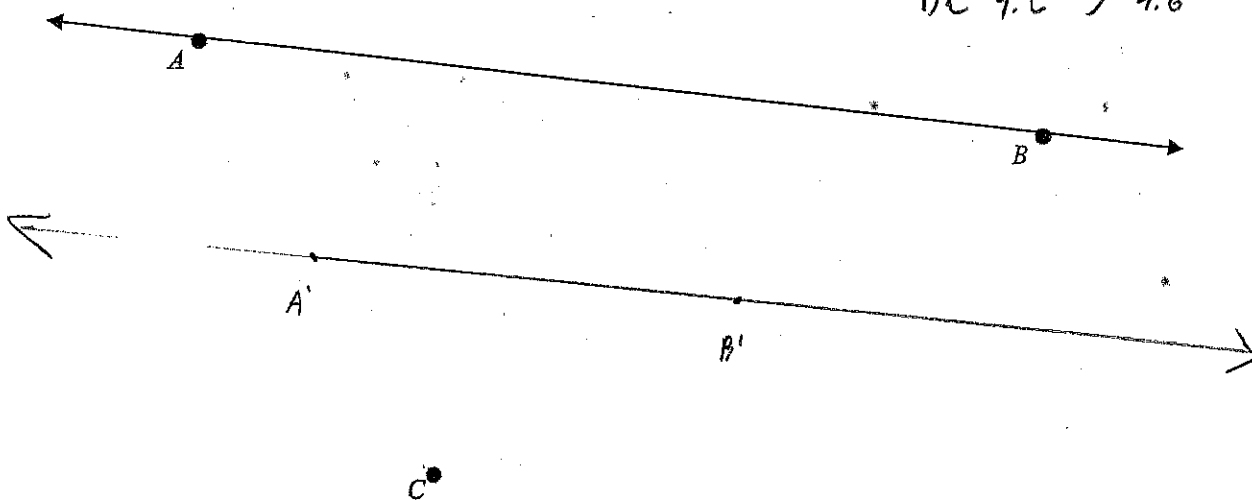
Dilation or not

If dilation will the image be:

Similar or not

7. Dilate the line \overline{AB} by a factor of 0.5 from point C.

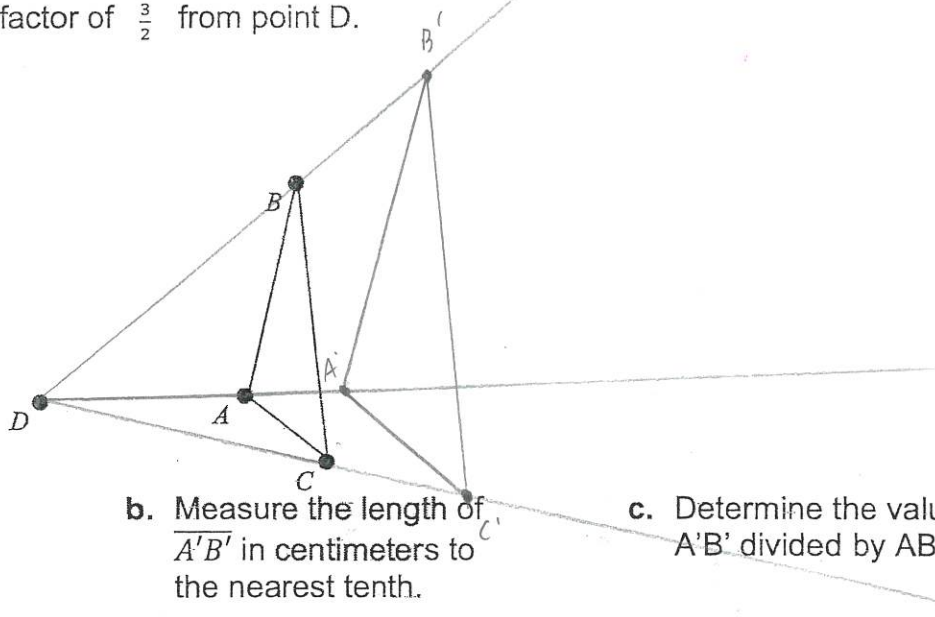
$CA 6.5 \rightarrow 3.25$
 $BC 9.2 \rightarrow 4.6$



How could you characterize the lines \overline{AB} and $\overline{A'B'}$? Parallel

$$\begin{aligned} DA & 2.7 \rightarrow 4.05 \\ DB & 4.5 \rightarrow 6.75 \\ DC & 3.8 \rightarrow 5.85 \end{aligned}$$

8. Dilate the $\triangle ABC$ by a factor of $\frac{3}{2}$ from point D.



- a. Measure the length of \overline{AB} in centimeters to the nearest tenth.

$$\overline{AB} = \underline{3}$$

- b. Measure the length of $\overline{A'B'}$ in centimeters to the nearest tenth.

$$\overline{A'B'} = \underline{4.5}$$

- c. Determine the value of $\overline{A'B'}$ divided by \overline{AB} .

$$\frac{\overline{A'B'}}{\overline{AB}} = \underline{1.5}$$

- d. What might you conclude about the scale factor and the ratio of dilated segment measure to its pre-image?

The same

- e. Measure angle $\angle BAC$ and the angle $\angle B'A'C'$ using the protractor.

$$m\angle BAC = \underline{115^\circ}$$

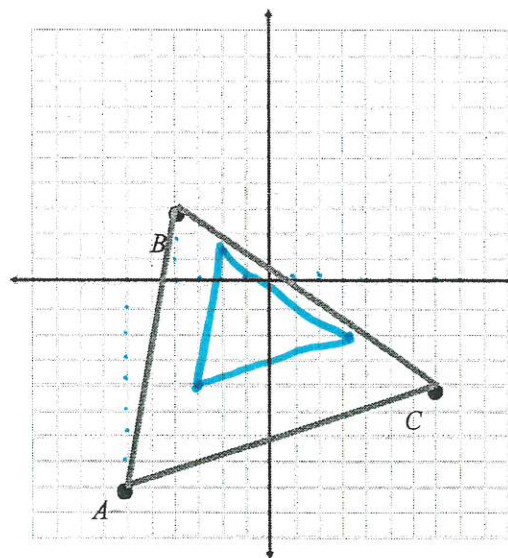
$$m\angle B'A'C' = \underline{115^\circ}$$

- f. What might you conclude about the each pair of corresponding angles?

congruent

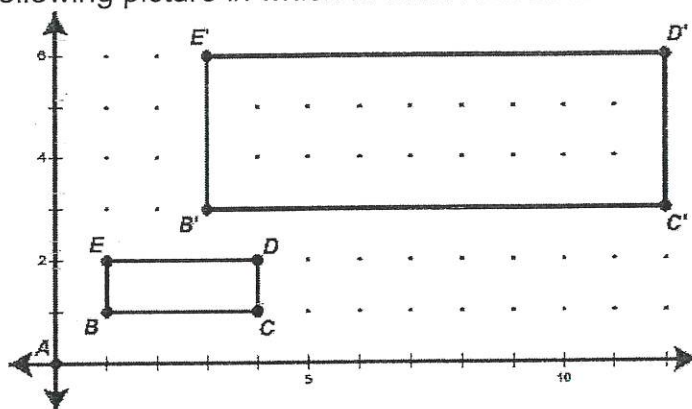
3. $\triangle ABC$ with center of dilation at origin and scale factor of $1/2$.

$$\begin{array}{ll} A(-6, -8) & A'(-3, -4) \\ B(-4, 3) & B'(-2, 1.5) \\ C(7, -4) & C'(3.5, -2) \end{array}$$



What do you notice about corresponding sides? proportional

9. Consider the following picture in which $\square BCDE$ has been dilated from point A.



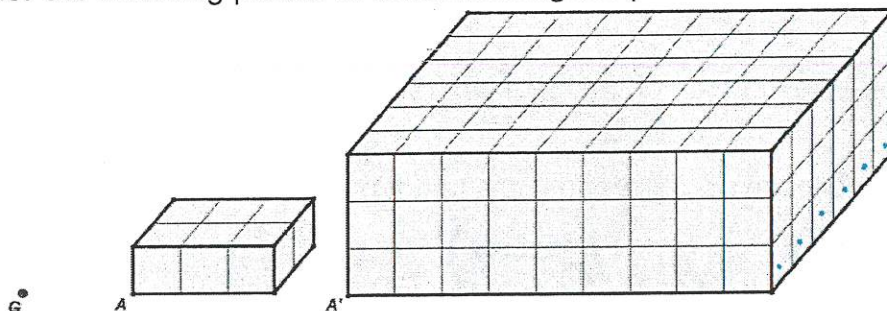
- a. What is the scale factor of the dilation based on the sides?
3
- b. What is the area of $\square BCDE$?
3
- c. What is the area of $\square B'C'D'E'$?
27

- d. What is the value of the area of $\square B'C'D'E'$ divided by the area of $\square BCDE$?
9

- e. What might you conclude about the ratio of two dilated shapes' side lengths compared to the ratio of their areas?

the ratio of the areas is the square of the ratios of the sides

10. Consider the following picture in which rectangular prism A has been dilated from point G.



- a. What is the scale factor of the dilation based on the sides?
3
- b. What is the volume of rectangular prism A?
6
- c. What is the volume of rectangular prism A'?
162

- d. What is the value of the volume of prism A' divided by the volume of prism A?
27

- e. What might you conclude about the ratio of two dilated solids' sides compared to the ratio of their volumes?

the ratio of the volumes is the cube of the ratios of the sides

Plot the following points and connect the consecutive points.

R (1,3)

S (2,2)

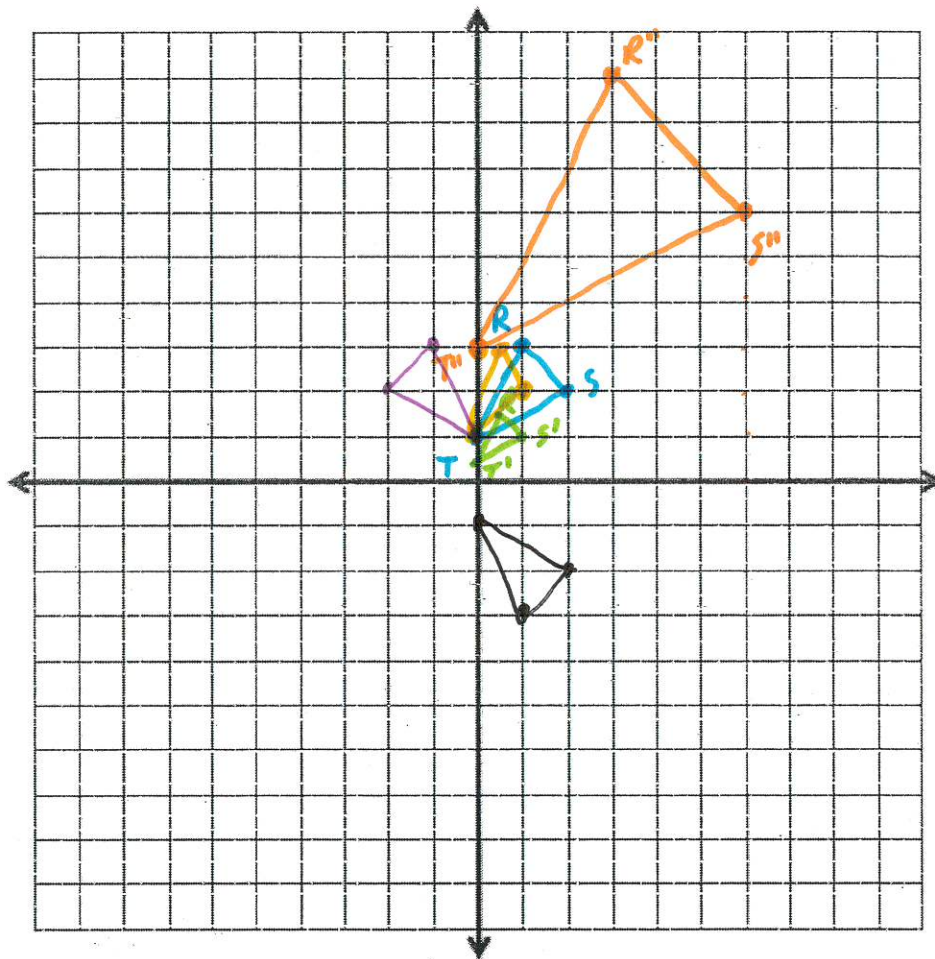
T(0,1)

12. Using a different color, dilate $\triangle RST$ using rule $D:(x,y) \rightarrow (0.5x, 0.5y)$.

$R' (0.5, 1.5)$

$S' (1, 1)$

$T' (0, 0.5)$



Use the rules given in #14-17 to create additional dilations. List the new coordinates and graph them on the coordinate plane above. Use a different color for each dilation. (Dilate your original points from #1 each time)

13. What would happen with the rule: $D:(x,y) \rightarrow (3x, 3y)$? $R(3, 9)$ $S(6, 6)$ $T(0, 3)$

14. What would happen with the rule: $D:(x,y) \rightarrow (0.5x, 1y)$? $R(0.5, 3)$ $S(1, 2)$ $T(0, 1)$

15. What would happen with the rule: $R:(x,y) \rightarrow (-1x, 1y)$? $R(-1, 3)$ $S(-2, 2)$ $T(0, 1)$

16. What would happen with the rule: $R:(x,y) \rightarrow (1x, -1y)$? $R(1, -3)$ $S(2, -2)$ $T(0, -1)$

17. Figure FCDE has been dilated to create F'C'D'E'.

- a. What is the dilation scale factor?

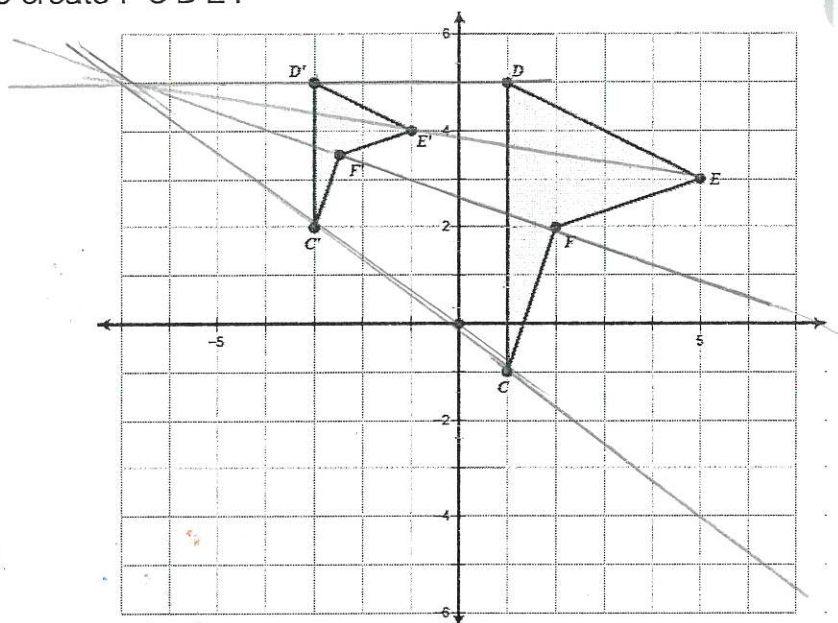
2

- b. What is the location of the center of dilation?

$(-7, 5)$

- c. What is the ratio of the areas?

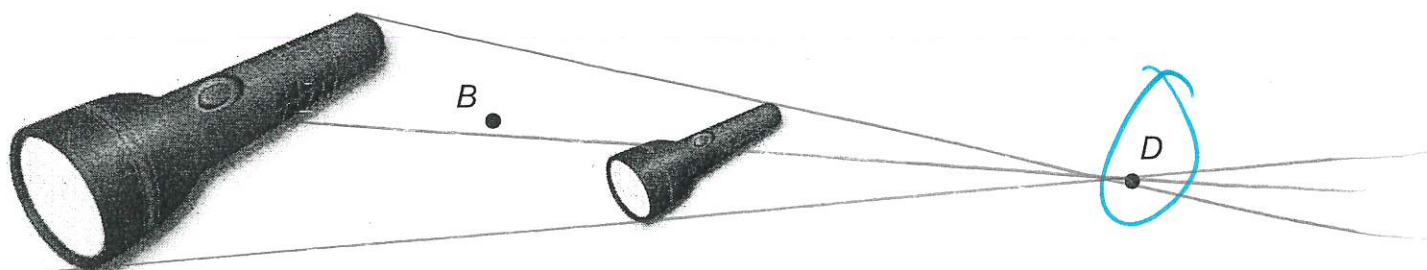
4

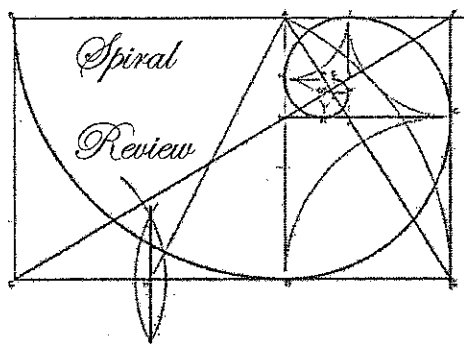


18. Which point would be the center of dilation?

A

C





The following problems will review previously learned skills. They are intended to keep skills fresh in your mind as well as foreshadow future lessons.

Solve for x . Show each step and circle your final answer.

1. $6n + 8 = 4n - 5 + 6n - 3$

$$8 = 4n - 8$$

$$16 = 4n$$

$$n = 4$$

2. $5(2x - 3) + 4x = 13$

$$10x - 15 + 4x = 13$$

$$14x - 15 = 13$$

$$14x = 28$$

$$x = 2$$

3. $5(x + 6) = 3(x + 1) + 5x$

$$5x + 30 = 3x + 3 + 5x$$

$$5x + 30 = 8x + 3$$

$$27 = 3x$$

$$x = 9$$

4. $8(x + 3) - 2x = 4 - 4(x + 10)$

$$8x + 24 - 2x = 4 - 4x - 40$$

$$6x + 24 = -4x - 36$$

$$10x + 24 = -36$$

$$10x = -60$$

$$x = -6$$

5. $\frac{x}{12} = \frac{1}{3}$

$$3x = 12$$

$$x = 4$$

6. $\frac{2}{3x+1} = \frac{1}{x}$

$$2x = 3x + 1$$

$$-x = 1$$

$$x = -1$$

Glossary

- **Angle:** A figure created by two distinct rays that share a common endpoint (also known as a vertex). $\angle ABC$ or $\angle B$ or $\angle CBA$ indicate the same angle with vertex B.
- **Angle of Rotation:** The amount of rotation (in degrees) of a figure about a fixed point such as the origin.
- **Bisector:** A point, line or line segment that divides a segment or angle into two equal parts.
- **Circle:** The set of all points equidistant from a point in a plane.
- **Congruent:** Having the same size, shape and measure. $\angle A \cong \angle B$ indicates that angle A is congruent to angle B.
- **Corresponding angles:** Angles that have the same relative position in geometric figures.
- **Corresponding sides:** Sides that have the same relative position in geometric figures.
- **Dilation:** Transformation that changes the size of a figure, but not the shape.
- **Endpoint:** The point at each end of a line segment or at the beginning of a ray.
- **Image:** The result of a transformation.
- **Intersection:** The point at which two or more lines intersect or cross.
- **Isometry:** transformation which preserves length and angle measures and are said to be geometrically congruent. Isometries include rotation, reflection, translations and glides
- **Line:** One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely. \overleftrightarrow{AB} indicates a line that passes through points A and B.
- **Line segment:** A part of a line between two points on the line. \overline{AB} indicates the line segment between points A and B.
- **Parallel lines:** Two lines are parallel if they lie in the same plane and do not intersect. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ indicates that line AB is parallel to line CD.
- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ indicates that line AB is perpendicular to line CD.
- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.
- **Pre-image:** A figure before a transformation has taken place.
- **Ray:** A part of a line that begins at a point and continues forever in one direction. \overrightarrow{AB} indicates a ray that begins at point A and continues in the direction of point B indefinitely.
- **Reflection:** A transformation of a figure that creates a mirror image, "flips," over a line.