Algebra 1 Unit 4
Characteristics of Functions

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This packet belongs to:

____

ALGEBRA

Georgia Milestones Assessment System

Algebra 1 Formula Sheet

Below are the formulas you may find useful as you take the test. However, you may find that you do not need to use all of the formulas. You may refer to this formula sheet as often as needed.

Linear Formulas

Slope Formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Linear Equations

Slope-intercept Form: $y = mx + b$

Point-slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By = C$

Arithmetic Sequence Formulas

Recursive: $a_n = a_{n-1} + d$

Explicit: $a_n = a_1 + (n-1)d$

Exponential Formulas

Exponential Equation

$y = ab^x$

Geometric Sequence Formulas

Recursive: $a_n = r(a_{n-1})$

Explicit: $a_n = a_1 \cdot r^{n-1}$

Compound Interest Formula

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Quadratic Formulas

Quadratic Equations

Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x-h)^2 + k$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Average Rate of Change

The change in the y-value divided by the change in the x-value for two distinct points on a graph.

Statistics Formulas

Mean

$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Interquartile Range

$IR = Q_3 - Q_1$

The difference between the first quartile and third quartile of a set of data.

Mean Absolute Deviation

$\frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$

The sum of the distances between each data value and the mean, divided by the number of data values.

Write your Questions here!

Whole Group Lesson

Introduction: The Parent Functions

Learning Targets:

- **I can identify the visual differences between linear, quadratic and exponential functions.**

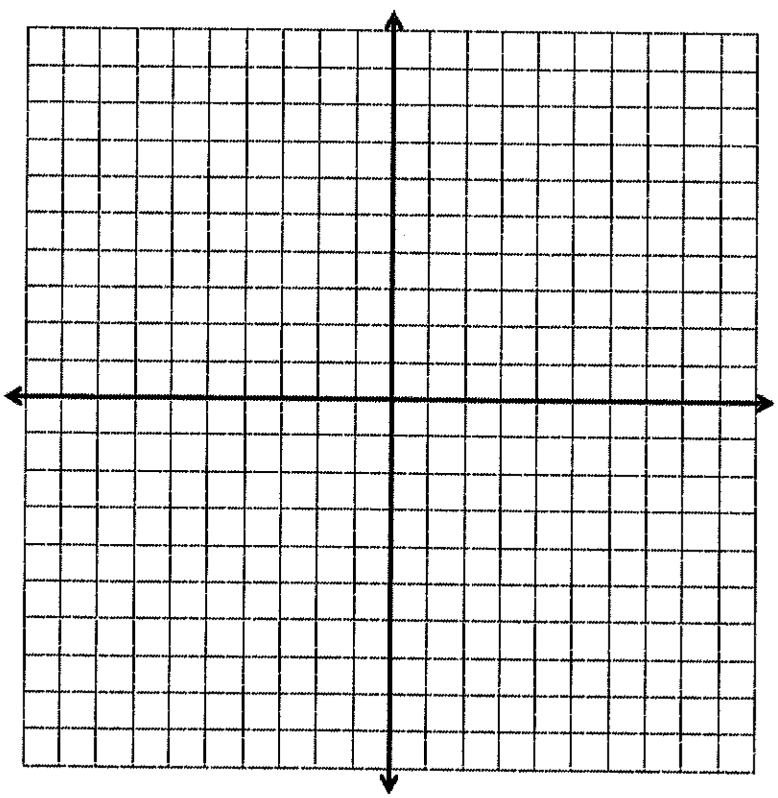
F.LE.3

No Video

Complete the table below.

Linear		**Quadratic**		**Exponential**	
$f(x) = x$		$g(x) = x^2$		$h(x) = 2^x$	
x	$f(x)$	x	$g(x)$	x	$h(x)$
-5		-5		-5	
-4		-4		-4	
-3		-3		-3	
-2		-2		-2	
-1		-1		-1	
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	

Draw and label each graph on the same coordinate plane.



Observations:

Linear

Quadratic

Exponential

Whole Group Lesson

Transformations

Vertical Shifts

Adding a value to the function (outside the parenthesis) shifts the graph _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Subtracting a value from the function (outside the parenthesis) shifts the graph _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Horizontal Shifts

Adding a value to the x-value (inside the parenthesis) shifts the graph _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Subtracting a value from the x-value (inside the parenthesis) shifts the graph _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Reflections

Multiplying the function by a negative _________ the graph across the _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Vertical Stretches and Shrinks

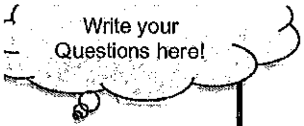
Multiplying the function by a value greater than 1 causes the graph to have a vertical _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Multiplying the function by a value less than 1 causes the graph to have a vertical _________.

Ex: $f(x) = 2^x$
$h(x) =$ _________

Write your Questions here!



4-1: Transformations

Learning Targets:

- **I can determine the effect transformations have on a parent graph.**

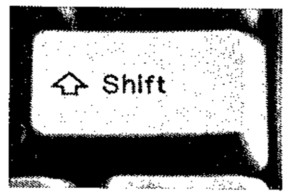
F.BF.3

Begin 4-1 Video 1

Vocabulary:
- Exponential
- Horizontal Shift
- Linear
- Quadratic
- Reflection
- Vertical Shift

In this course we focus on three types of functions. Linear, Quadratic and Exponential. In the intro lesson, you saw what each of these three parent, or basic, functions look like. Now we are going to look at what happens when you change that function. Transformations of functions are very similar to what you do in Geometry when moving figures around the coordinate plane.

In this lesson, any reference to k, is a positive real number.



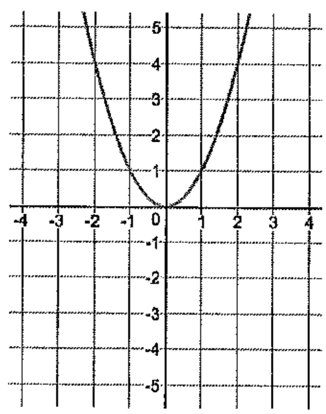
Vertical Shifts (up & down)

A shift may sometimes be referred to as a _______________. If k is **added** to the _______________ where we go from $f(x)$ to $f(x) + k$, then the graph of $f(x)$ will vertically shift _____________ by k units. If k is **subtracted** from the _______________ where we go from $f(x)$ to $f(x) - k$, then the graph of $f(x)$ will vertically shift _________________ by c units.

In general, a vertical translation means that every ordered pair will go from (x, y) on the original $f(x)$ to $(x, y + k)$ on the newly transformed $f(x) + c$ or to $(x, y - k)$ on the newly transformed $f(x) - k$.

Example 1:

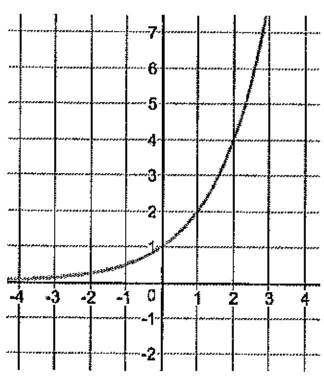
1a) Write an equation that would transform the parent function $f(x) = x^2$ up 3 units.



1b) Write an equation that would transform the parent function $f(x) = x^2$ down 4 units.

Example 2

2a) What would happen to the graph of $g(x) = 2^x$ if we transformed it to be $h(x) = 2^x - 1.5$?

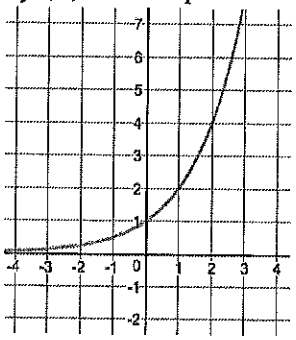


2b) What would happen to the graph of $g(x) = 2^x$ if we transformed it to be $h(x) = 2^x + 3$?

Write your Questions here!

You try:

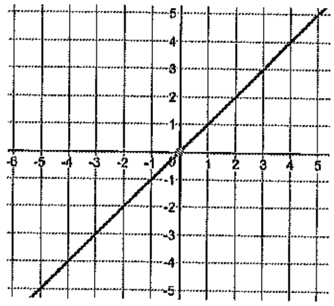
1. Write an equation that would transform the parent function $f(x) = 2^x$ up 4 units.



$f(x) = 2^x + 4$

2. What would happen to the graph of $y = x$ if we transformed it to be $y = x - 2$?

shift up by 2



Begin 4-1 Video 2

Horizontal Shifts (left & right)

If *c* is **added** to the________________ of the function where we go from *f(x)* to *f(x + c)*, then the graph of *f(x)* will be horizontally shifted to the____________ *c* units. If *c* is **subtracte**d from the_________________of the function where we go from *f(x)* to *f(x – c)*, then the graph of *f(x)* will be horizontally shifted to the___________*c* units.

Notice that anytime we are making a change "inside the parenthesis", things shift in the opposite direction of what you would expect.

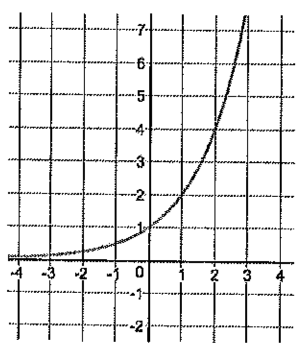
- Add to go________.
- Subtract to go_______________.

Adding C moves the function to the **left** (the negative direction).

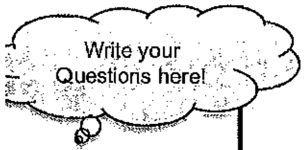
Why? Well imagine you will inherit a fortune when your **age=25.** If you change that to **(age+4) = 25** then you will get it when you are 21. Adding 4 made it happen earlier.

Example 3:

3a) Write an equation that would transform the parent function $y = 3^x$ to the left 2 points.

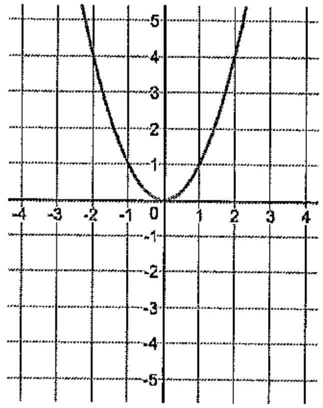


3b) Write an equation that would transform the parent function $y = 3^x$ to the right 3 points.



Example 4:

4a) What would happen to the graph of $g(x) = x^2$ if we transformed it to be $h(x) = (x + \frac{1}{3})^2$?



4b) What would happen to the graph of $g(x) = x^2$ if we transformed it to be $h(x) = (x - 3)^2$?

You try:

1. Write an equation that would transform the parent function $f(x) = x^2$ right 1 unit.

$y = (x-1)^2$

2. What would happen to the graph of $g(x) = 2^x$ if we transformed it to be $h(x) = 2^{x+4}$?

Shift left 4

Begin 4-1 Video 3

Vertical stretching and shrinking

If c is____________________ to the function then the graph of the function will undergo a vertical _______________ or__________________.

- When the function becomes $y = c * f(x)$ and $0 < c < 1$, a vertical _______________ happens. This fractional c pulls the y values down towards the x-axis.
- When $c > 1$ in the function $y = c * f(x)$, a vertical_______________ occurs. The c multiplies the y-values and pushes them up and away from the x-axis.

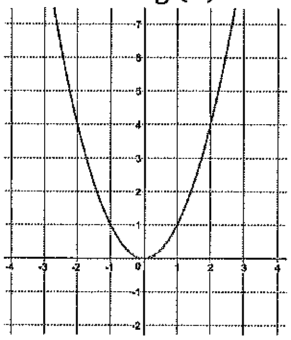
In general, a vertical stretching or shrinking means that every point (x, y) on the graph is transformed to (x, cy) on the new graph.

When you vertically stretch a graph it is going to look tall and skinny.
When you vertically shrink a graph, it will look short and wide.
To make sense of this, think of stretching or compressing a rubber band!

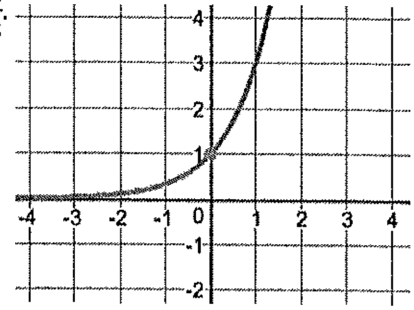
Write your Questions here!

Example 5:

5a) Describe the transformation that would happen to the graph of $f(x) = x^2$ if it was multiplied by a factor of 4 such that $g(x) = 4x^2$.



5b) Write an equation that would vertically shrink the parent function $f(x) = 3^x$ by a factor of $\frac{1}{2}$.



You try:

1. Write an equation that would vertically shrink the parent function $f(x) = x^2$ by a factor of $\frac{1}{2}$.

$f(x) = \frac{1}{2}x^2$

2. Describe the transformation that would happen to the graph of $f(x) = 3^x$ if it was multiplied by a factor of 2 such that $g(x) = 2 * 3^x$.

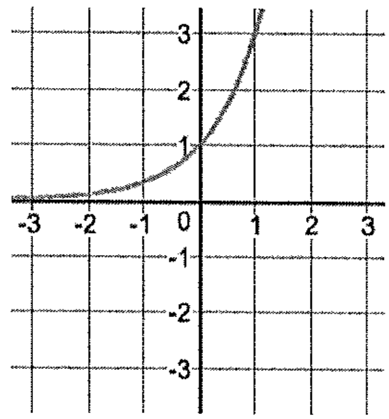
Stretch by a factor of 2

Begin 4-1 Video 4

Reflection

If the function is multiplied by -1, the graph of the function will undergo a reflection over the x-axis.

Example 6: Using the graph provided, show that multiplying the function $f(x) = 3^x$ by -1 will reflect the graph across the x-axis.



Example 7: Describe how the graph of $f(x) = 4^x$ has been transformed to form the graph of $h(x) = -3(4)^{(x-2)} + 6$

-
-
-
-

You try:

1. Describe the transformation(s) on each function.

a. $f(x) = \frac{1}{5}(2)^x - 4$

- shrink by a factor of $\frac{1}{5}$
- Shift down 4

b. $f(x) = 5(2)^{x-3}$

- Stretch by a factor of 5
- Shift right 3

c. $y = -2(x+4)^2 + 4$

- stretch by a factor of 2
- reflect over x-axis
- Shift left 4
- Shift up 7

Write your Questions here!

Practice Makes Better

Practice 4-1: Transformations
Describe the transformation on each function.

1. $f(x) = 5x$
   - o Stretch by 5

2. $f(x) = -x + 3$
   - o reflect over x-axis
   - o Shift up 3

3. $f(x) = -\frac{1}{2}x + 4$
   - o reflect over x-axis
   - o Shrink by 1/2
   - o Shift up 4

4. $f(x) = 2x - 5$
   - o stretch by 2
   - o Shift down 5

5. $f(x) = -\frac{1}{5}x - 2$
   - o reflect over x-axis
   - o Shrink by 1/5
   - o Shift down 2

6. $f(x) = 5x + 3$
   - o Stretch by 5
   - o Shift up 3

7. $f(x) = \frac{1}{2}x + 4$
   - o Shrink by 1/2
   - o Shift up 4

8. $f(x) = -\frac{2}{5}x - 4$
   - o reflect over x-axis
   - o Shrink by 2/5
   - o Shift down 4

9. $f(x) = 2(x - 1)^2 - 2$
   - o Stretch by 2
   - o Shift right 1
   - o Shift down 2

10. $f(x) = 2x^2 + 3$
    - o Stretch by 2
    - o Shift up 3

11. $f(x) = \frac{1}{2}(x + 2)^2$
    - o Shrink by $\frac{1}{2}$
    - o Shift left 2

12. $f(x) = -\frac{2}{3}(x)^2 - 4$
    - o reflect over x-axis
    - o Shrink by 2/3
    - o Shift down 4

13. $y = -(x - 3)^2 + 2$
    - o reflect over x-axis
    - o Shift right 3
    - o Shift up 2

14. $y = -4(2)^{x+2} + 3$
    - o reflect over x-axis
    - o Shift left 2
    - o Shift up 3

15. $y = \frac{3}{4}(x - 3)^2 - 4$
    - o Shrink by 3/4
    - o Shift up 4
    - o Shift right 3

16. $f(x) = 3x^2 - 3$
    - o Stretch by 3
    - o Shift down 3

17. $f(x) = \frac{1}{3}(2)^x + 2$
    - o Shrink by 1/3
    - o Shift up 2

18. $f(x) = 2(2)^x - 1$
    - o Stretch by 2
    - o Shift down 1

19. $f(x) = 2^{x-2}$
    - o Shift right 2

20. $f(x) = -2^x + 3$
    - o reflect over x-axis
    - o Shift up 3

Write your Questions here!

Write the equation of a function that has undergone the following transformations.

21. A linear function that has been shifted to the right two.

$f(x) = (x-2)$

22. A quadratic function that has been stretched by a factor of two and shifted down by four.

$f(x) = 2x^2 - 4$

23. An exponential that has been reflected over the x-axis and shifted up 2.

$f(x) = -2^x + 2$

24. A quadratic function that has been shrunk by a factor of $\frac{1}{2}$, shifted right 4, and up 3.

$f(x) = \frac{1}{2}(x-4)^2 + 3$

25. An exponential function that has been stretched by a factor of 2, reflected over the x-axis, shifted down two, and left one.

$f(x) = -2(2)^{x+1} - 2$

Spiral Practice

This WILL be on your mastery check!

26. What is the area of a square with the length of a side equaling $3a^5$?

$9a^{10}$

27. What is the area of the rectangle with the width of $6x^2$ *and the length of* $12x^3$?

$72x^5$

28. How many one-third cup servings are in 6 cups of pecans?

18 cups

29. The perimeter of an isosceles triangle is 16 inches. The length of its base is 2 times the length of one of its other sides. In the equations below, x represents the length of each of its equal sides and y represents the length of its base. What are the three side lengths of the triangle?

$2x + y = 16$ and $y = 2x$

base = 8
equal sides = 4

Review your practice and notes to prepare for the mastery check.

Write your Questions here!

4-2: Rate of Change

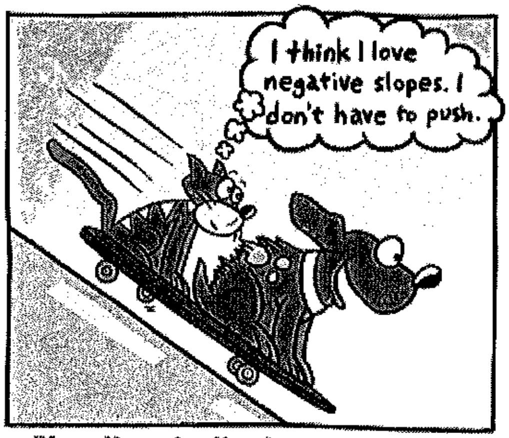
Learning Target: I can find and interpret the rate of change of linear, exponential and quadratic functions.

F.IF.5

Vocabulary:
- Average Rate of Change
- Coefficient
- Constant Rate of Change
- Slope

Begin 4-2 Video 1

The ____________________, or the ____________, is a ratio describing how one quantity changes as another quantity changes. In algebra, we are typically comparing how the output changes to how the input changes.



"I estimate that we are on a slope of about −0.625. What do you think?"

Average rate of change

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Formula Sheet!

If the value of the slope is ____________, then we know that the rate of change is increasing over time.

If the rate of change ___________ over time, you should expect a negative value.

There are many different ways you may be asked about rate of change.

Example 1 - From ordered pairs:

1a) Find the slope of the line that goes through the points (–4,2) and (–6,4).

1b) Find the slope of the line that goes through the points (3,5) and (3,2).

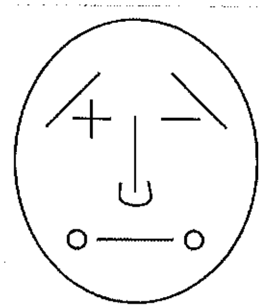
You try:

1. Find the slope of the line that goes through the points (3,–1) and (–2,5).

$-\frac{6}{5}$

2. Find the slope of the line that goes through the points (3,2) and (1,2).

0



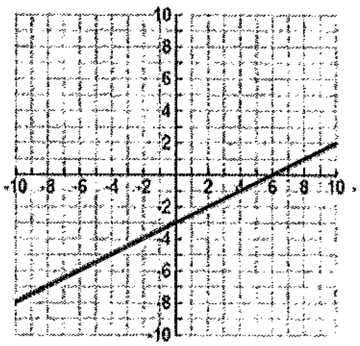
MR. SLOPE GUY

Write your Questions here!

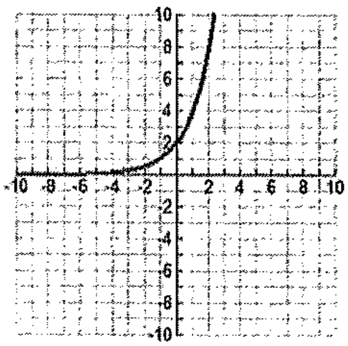
Begin 4-2 Video 2

Linear functions have a _______ rate of change, meaning values increase or decrease at the same rate over a period of time. This also means that no matter where you look on the graph, the rate of change will be exactly the same.

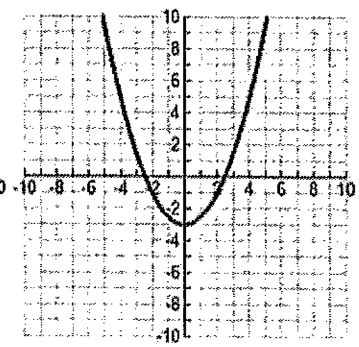
Both Exponential and Quadratic have _____________ rates of change. This means that the graph of the function will be steeper in some places than others. When calculating average rate of change for these functions, you will need to specify an interval.



Linear



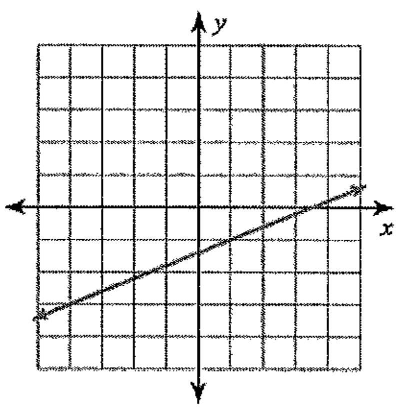
Exponential



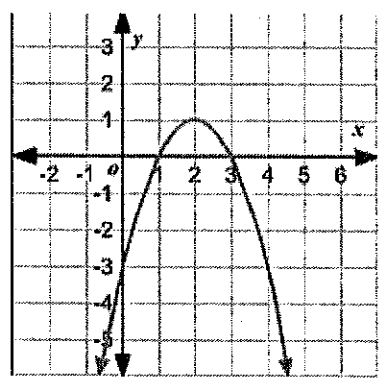
Quadratic

Example 2 - From a graph:

2a) Determine the slope of the line.

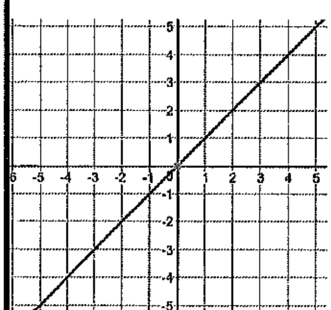


2b) Determine the average rate of change over the interval [0, 2].



You try:

1. Determine the slope of the line.



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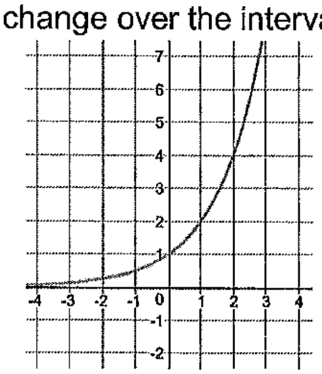
2. Determine the average rate of change over the interval [1, 2].



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Write your Questions here!

Begin 4-2 Video 3

Example 3 – from an equation:

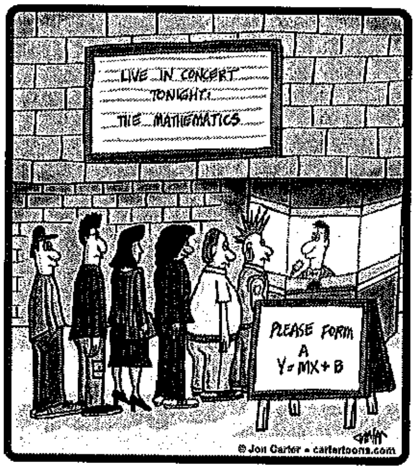
3a) Find the slope of the linear function, $3x - 2y = 8$.

3b) Determine the average rate of change from $x = 1$ to $x = 2$,given the function,$f(x) = x^2 + 2x - 7$.

You try:

1. Find the slope of the line, $y = -\frac{4}{3}x + 3$.

$-\frac{4}{3}$



2. Given the function, $f(x) = 3^{x+1}$, determine the average rate of change from $x = -1$ to $x = 1$.

4

Begin 4-2 Video 4

Example 4 – from a table:

Given the table of values for $h(x)$, what is the average rate of change from $x = 6$ to $x = 2$?

x	– 2	0	2	4	6
$h(x)$	3	– 1	3	15	35

Example 5 – from a scenario:

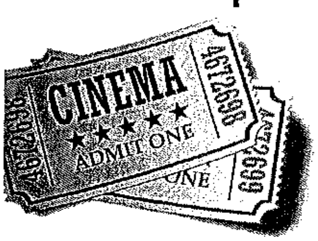
You are trying to decide which theater to go to for a Friday night movie. Habersham Hills charges $7 for the movie ticket and $5 per food item. Regal's prices are represented by the table.

Write an equation for Habersham Hills and Regal. Compare their rates of change and initial cost.

Number of food items with ticket (x)	Price in $ (y)
0	8
1	12
2	16
3	20
4	24

Habersham Hills:

Regal:



Which theater is cheaper if you want to see the movie and also get a drink and popcorn?

Write your Questions here!

You try:

1. The average price for a ticket to a movie theater in North America for selected years is shown in the table below.
Find the rate of change from 2010 to 2015. Then find the rate of change from 1995 to 2005? How do the rates of change compare?

Year	1985	1990	1995	2000	2005	2010	2015
Price ($)	3.90	4.20	4.50	4.80	5.10	5.40	5.70

$\frac{.30}{5} = \$.06/\text{yr}$ $\frac{.6}{10} = \$.06/\text{yr}$

2. Given the table of values for $f(x)$, what is the average rate of change from $x = 1$ to $x = 3$?

x	−1	0	1	2	3
$f(x)$	−0.25	−1	−4	−16	−64

−30

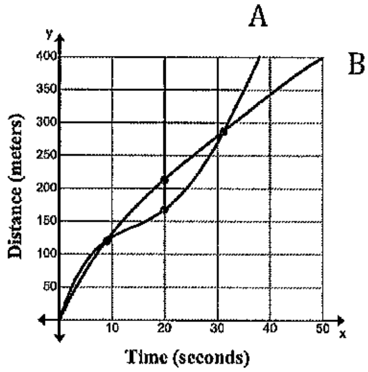
Write your Questions here!

Practice Makes Better

Practice 4-2: Rate of Change

Use the information provided to answer the questions below.

1. Below is the graph and table for 2 runners running the 400 meter hurdles race.



Time	Runner A	Runner B
0	0	0
9	120	120
20	168	213
31	287	287

a) Which runner has a faster average speed for the first 9 seconds? same speed

b) Which runner has a faster average speed from 9 to 20 seconds? B

c) Which runner has a faster average speed from 20 to 31 seconds? A

d) Which runner has a faster average speed from 9 to 31 seconds? same

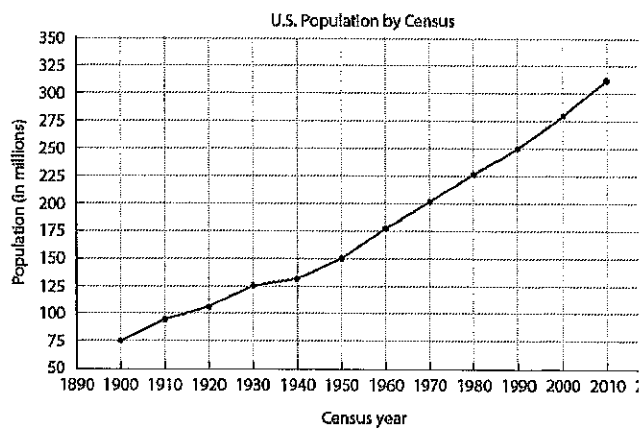
Hint: Speed is another name for average rate of change!

e) Which runner wins the race? A
How do you know? reaches 400 1st

2. The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.

a) What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

2.05 mil/yr
3.05 mil/yr
less



b) Which 10-year time periods have the highest and the lowest rates of change? How did you find these?

1990-2000 & 2000-2010

vary

3. Find the rate of change of Pete's height from 3 to 5 years.

Time (years)	1	2	3	4	5	6
Height(in.)	27	35	37	42	45	49

4in/yr

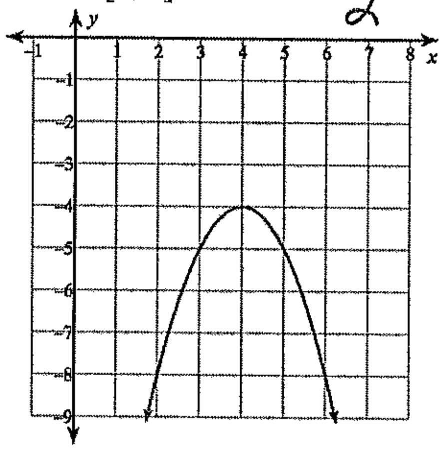
4. For $f(x) = -6x - 2$, find the rate of change on the interval [-2, 4].

-6

Find the rate of change (slope) of each graph over the given interval.

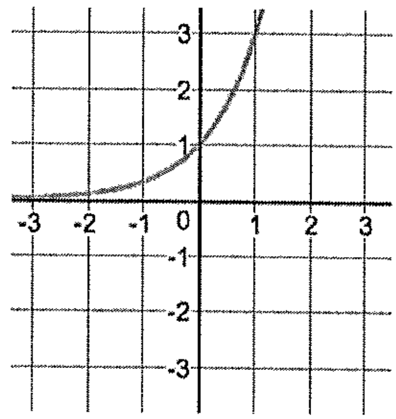
5. Find the rate of change over the interval [2, 4]

2

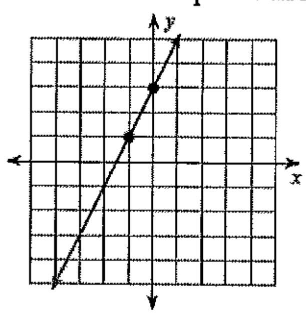


6. Find the average rate of change over the interval [0,1]

2

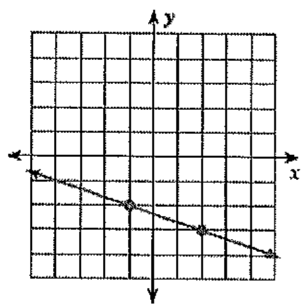


7. Find the slope of the line.



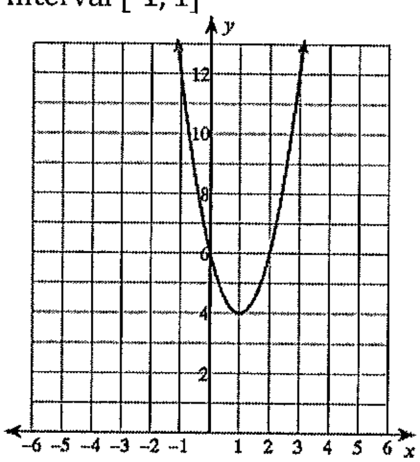
2

8. Find the slope of the line.



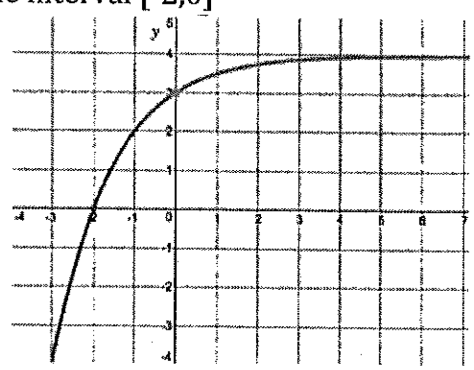
$-1/3$

9. Find the rate of change over the interval [-1, 1]



-4

10. Find the average rate of change over the interval [-2,0]



3/2

Write your Questions here!

Find the rate of change (slope) through each pair of points.

11. (-14, -8), (-10, -7)

1/4

12. (5, 4), (8, 5)

1/3

13. (-18, -17), (-17, 16)

33

14. (13, 7), (13, 10)

undefined

Find the rate of change (slope) of each line.

15. $y = -5x + 5$

-5

16. $y = 5x + 5$

5

17. $3x + y = 0$

-3

18. $x = 4$

undefined

Find the rate of change (slope) of each function over the given interval.

19. Determine the average rate of change from $x = -2$ to $x = 1$, given the function, $f(x) = x^2 + 4x - 3$.

3

20. Determine the average rate of change from $x = 0$ to $x = 2$, given the function, $f(x) = 2(3)^x - 1$.

3

21. Determine the average rate of change from $x = 0$ to $x = 2$, given the function, $f(x) = 5^x - 2$.

12

22. Determine the average rate of change from $x = -1$ to $x = 1$, given the function, $f(x) = x^2 - 6x + 9$.

-6

23. Given the table of values for $f(x)$, what is the average rate of change from $x = -2$ to $x = 1$?

x	−2	-1	0	1	2
f(x)	9	4	1	0	1

-3

24. Given the table of values for $g(x)$, what is the average rate of change from $x = 0$ to $x = 4$?

x	−2	0	2	4	6
g(x)	21	7	9	27	61

5

Write your Questions here!

25. Given the table of values for $h(x)$, what is the average rate of change from $x = 5$ to $x = 9$?

x	1	3	5	7	9
$h(x)$	-4	6	24	50	84

15

26. Given the table of values for $j(x)$, what is the average rate of change from $x = -4$ to $x = -2$?

x	−5	−4	−3	−2	−1
$j(x)$	3	0	-1	0	3

0

27. Find the rate of change of the cost per minute.

Minutes (x)	Cost in Dollars (f(x))
0	110.00
30	120.50
60	131.00
90	141.50
120	152.00

$0.35/min

28. Find the rate of change of revolutions per second.

Seconds (x)	Revolutions (f(x))
0	0
2	6
4	12
10	30
15	45

3 rev/sec

29. For the following two functions, write the equations of each and complete the chart using <, >, or = to compare them.

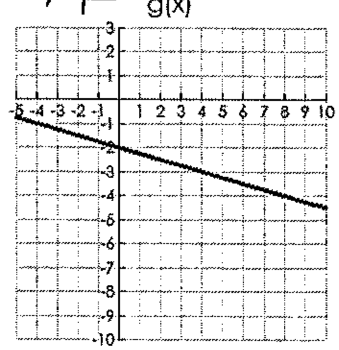
Characteristic of f(x)	<, >, or =	Characteristic of g(x)
y-intercept of f(x) = 5	>	y-intercept of g(x) = 2
f(4) = -3	=	g(4) = -3
Rate of change of f(x) = -2	<	Rate of change of g(x) = $-\frac{1}{4}$

x	f(x)
-3	11
-1	7
1	3
3	-1
5	-5

f(x) = $-2x+5$

g(x) = $-\frac{1}{4}x+2$

g(x)



Spiral Practice

This WILL be on your mastery check!

30. A pet store had 4 cats to feed. If they only had one-fifth of a bag of cat food and each cat got the same amount, what fraction of the bag would each cat get?

$\frac{1}{20}$

31. Solve for x in the following equation $5(x+2)=\frac{3}{5}(5+10x)$

$x=7$

32. Kaya and Tad started with the same number of baseball cards in their collections. Kaya collected 3 cards per week and now has 29 cards. Tad collected 2 cards per week and now has 20 cards. Let x represent the number of cards they began with, and let y represent the number of weeks. Write a linear system that represents this situation. How many weeks did they collect cards?

9 weeks

Review your practice and notes to prepare for the mastery check.

Write your Questions here!

4-3: Domain and Range

Learning Target:

- **I can identify the domain and range of a function.**

F.IF.1

Begin 4-3 Video 1

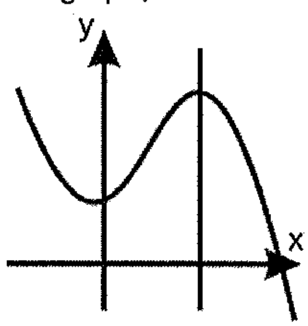
Vocabulary:
- Continuous
- Discrete
- Domain
- Interval Notation
- Range

The relationship between two quantities can be shown by a set of ordered pairs called a ________________.

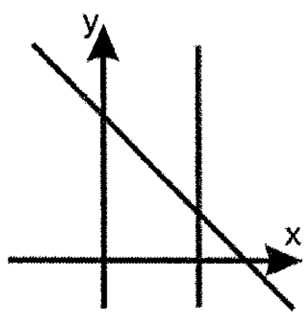
In the very first lesson of the course, we talked about the fact that a function was a special relation in which each input (x) has exactly ________output (y). The majority of the relations we will deal with throughout the remainder of the year will be functions. But how do you know if you're dealing with a function or just a relation?

Two ways to determine a function:

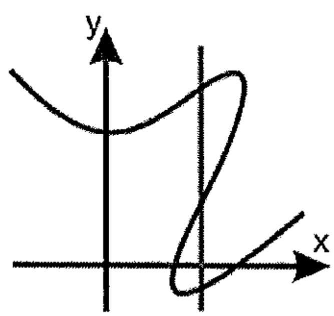
1. Scan the ______________
   - If an x value repeats, you ________________a function!
2. Vertical Line Test
   - If any _____________ line passes through more than one point of the graph, then the relation is **not** a function.



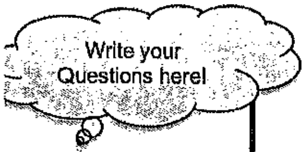
This is a function



This is a function



This is NOT a function

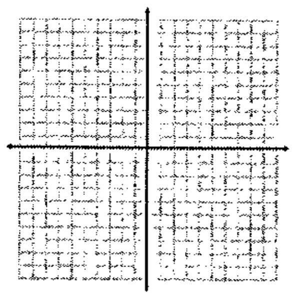


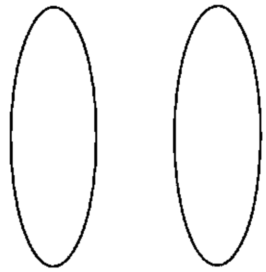
Relations can be expressed in several different ways.

The relation {(2, 3), (4, 7), (6,8)} can be displayed in the following ways:

An x/y table | A graph | Using Mapping

X	Y





The ____________of a relation is the set of inputs (or x-values) of the ordered pairs.

- "What x-values were used in this graph? How far left does it go? How far right?"

The__________ of a relation is the set outputs (or y-values) of the ordered pairs.

- "What y-values were used in this graph? How far down does it go? How far up?"

Example 1:

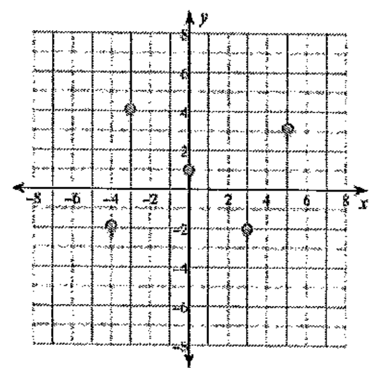
$\{(-1,4),(2,6),(0,4),(-7,1)\}$

Domain:

Range:

Function?

Example 2:



Domain:

Range:

Function?

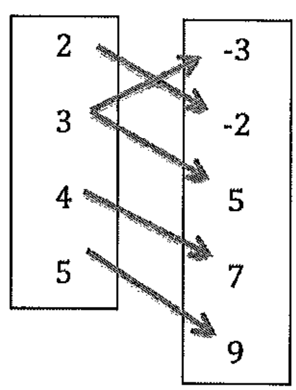
Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-values	y-values
Domain	Range
Input	Output
x	$f(x)$

Example 3:

Find the domain and range of the following relation.



Domain:

Range:

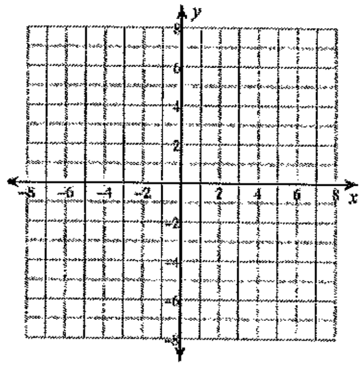
Function?

Write your Questions here!

Example 4:

Students who take the AP Government exam can score a 1, 2, 3, 4, or 5. Last year, two students scored a 1, no students scored a 2, three students scored a 3, seven students scored a 4 and seven students scored a 5. Write these scores as a set of ordered pairs and plot them on the coordinate plane.

Ordered Pairs:



Domain:

Range:

Function?

Should we connect these points with a line?

You try:

Identify the domain and range of each relation.

1.

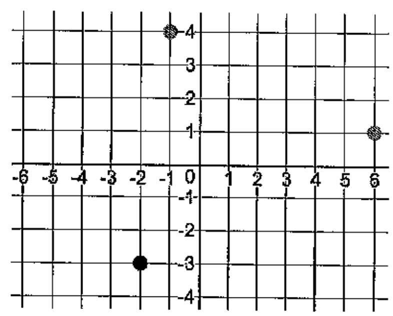
x	3	-1	-6	1	4
y	-4	7	-8	11	13

Domain:
x: {3, -1, -6, 1, 4}

Range:
y: {-4, 7, -8, 11, 13}

Function? yes

2.



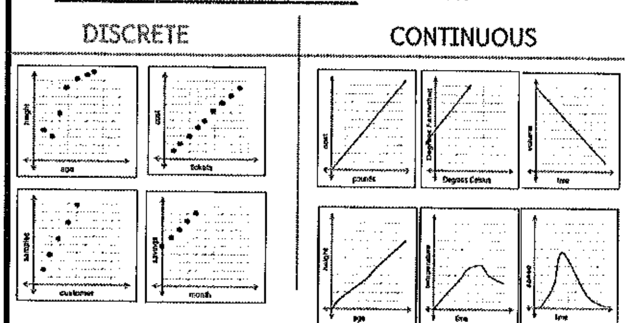
Domain:
{-2, -1, 6}

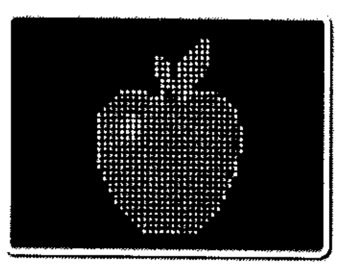
Range:
{-3, 4, 1}

Function?

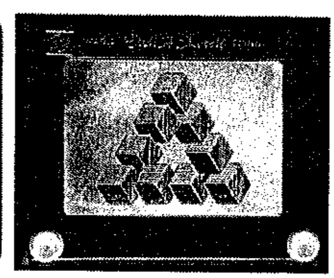
Begin 4-3 Video 2

All of the previous examples have had distinct points. We call relations like this ____________. But what if a relation has an infinite number of points? This is called a __________________ relation.





LITE BRITE
discrete



ETCH-A-SKETCH
continuous

There are a couple of different notations you can use to describe domain and range of continuous relations.

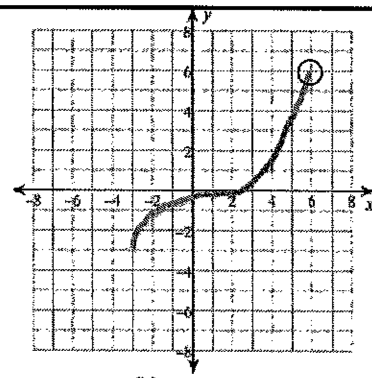
Set Notation

- Uses inequality symbols like <,>,≤, and ≥.
- When a boundary is included in the domain or range, we use either the ___________symbol.
  - Why? Because our domain or range can equal that value!
- If the boundary is not included in the domain or range, we use either the ________ symbol.
  - Why? Because the domain or range cannot equal that value.
- When either ____________________ is our bounds, we use ____________.
  - Why? You can never actually reach infinity!

Interval Notation

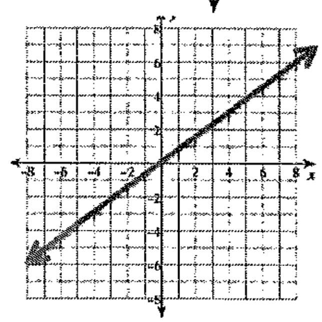
- Uses brackets and parentheses.
- We use brackets when the boundary is ______________.
- We use parentheses when the boundary is ______________.
- Use parentheses when a boundary is __________ or ____________________.
  - Why? You can never actually reach infinity!

Example 5:
Give the Domain and range of this continuous relation in set and interval notation.



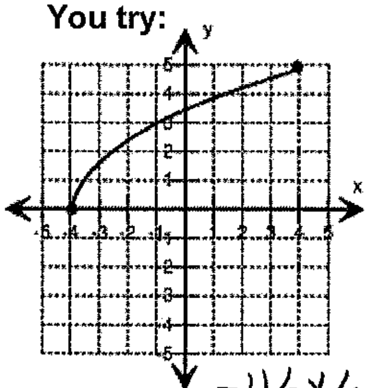
	Set Notation	Interval Notation
Domain		
Range		

Example 6:
Give the Domain and range of this continuous relation in set and interval notation.



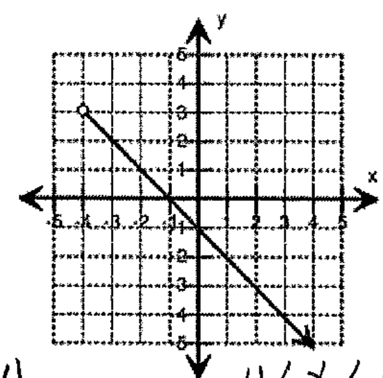
	Set Notation	Interval Notation
Domain		
Range		

You try:



Domain: $-4 \le x \le 4$

Range: $0 \le y \le 5$



Domain: $-4 < x < \infty$

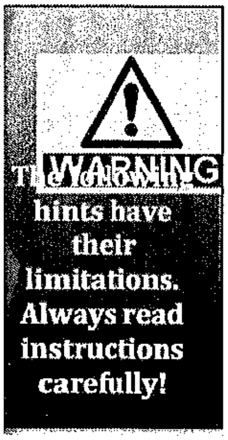
Range: $-\infty < y < 3$

How many ways can we say the same thing?

1. $-\infty < x < \infty$
2. All real numbers
3. $(-\infty, \infty)$
4. $\mathbb{R}$

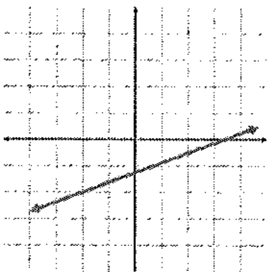
Write your Questions here!

Begin 4-3 Video 3



Math is all about patterns... and the three functions we focus on in Algebra 1, (Linear, Quadratic and Exponential) have some great patterns that make domain and range so much easier to deal with!

Linear:



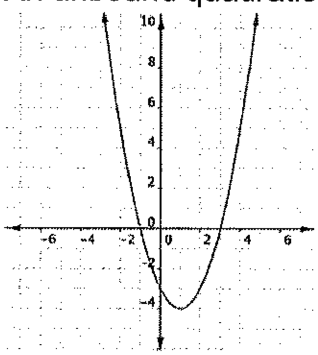
An unbound (meaning no restrictions) linear function will __________ have a domain of ℝ and a range of ℝ.

- A domain of all real numbers means that you can plug in any number you can think of and the function will give you an answer.
- A range of all real numbers means that eventually, your answers (outputs) will hit every number you can think of.

Example 7: Domain: ______________ Range: ______________

Quadratic:

An unbound quadratic function will have a domain of ℝ, the range will be restricted based upon the ____________.

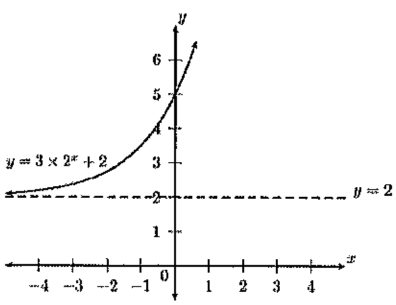


- Domain of all real numbers = plug in anything you want!
- The range (or possible outputs) of a quadratic function will be limited by the minimum or maximum value (vertex). This is the point at which the parabola ________. We will study these in more detail in a later lesson.

Example 8: Domain: ______________ Range: ______________

Exponential:

An unbound exponential function will have a domain of ℝ. The range will depend on the ________________.

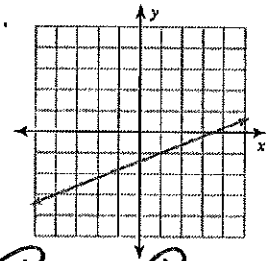


- Domain of all real numbers = Every x is a possibility!
- The range in exponentials is restricted because these functions have line that they cannot cross. This line is called an asymptote and changes from one function to another. We will study these in more detail in a later lesson.

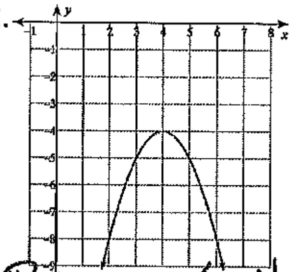
Example 9: Domain: ______________ Range: ______________

Write your Questions here!

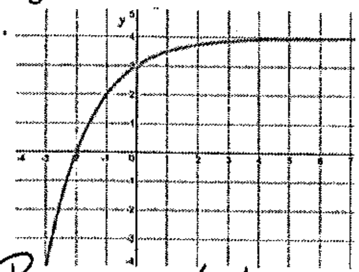
You try: Find the domain and range of each of the following functions.

1. 

D: $\mathbb{R}$ R: $\mathbb{R}$

2. 

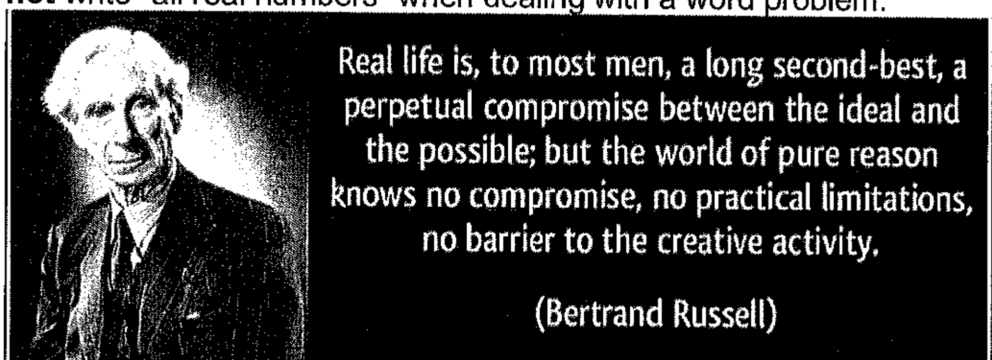
D: $\mathbb{R}$ R: $y \leq -4$

3. 

D: $\mathbb{R}$ R: $y < 4$

Begin 4-3 Video 4

Those patterns are awesome. But real life has limitations. Be prepared to **not** write "all real numbers" when dealing with a word problem.



Historical Note:
Bertrand Russell was a British philosopher and mathematician that was born in 1872 and died in 1970.

Example 10
Zach is making friendship bracelets to fund his spring break plans. He is renting a booth at the Mt. Laurel Festival to sell them and is charging $6 per bracelet. Zach's profit, y, in dollars, for selling x bracelets can be modeled by the function $y = 6x - 225$. What is the domain of this function?

Example 11
A tub drains at a rate of 3 gallons every 20 seconds. The typical tub holds approximately 30 gallons of water. Let y be the amount of water in the tub for a given x amount of time in seconds. Describe the domain and range of the function.

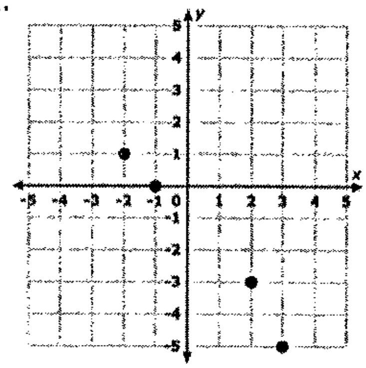
Write your Questions here!

Practice Makes Better

Practice 4 – 3: Domain & Range

Find the domain and range for each relation and determine whether it is a function.

1.

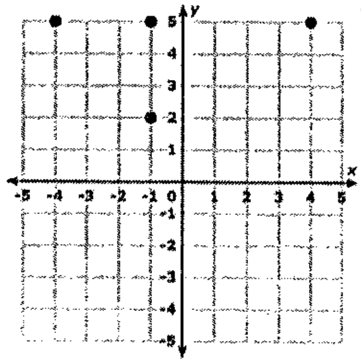


Domian: $\{-2,-1,2,3\}$

Range: $\{-5,-3,-1,1\}$

Function? yes

2.

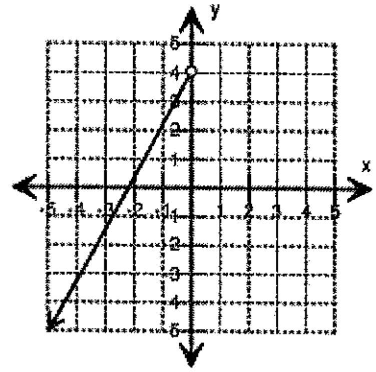


Domian: $\{-4,-1,4\}$

Range: $\{2,5\}$

Function? no

3.

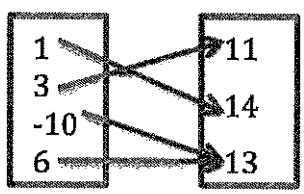


Domian: $\infty < x < 0$

Range: $-\infty < y < 4$

Function? yes

4.

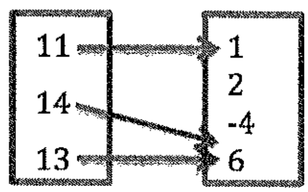


Domian: $\{-10,1,3,6\}$

Range: $\{11,13,14\}$

Function? yes

5.

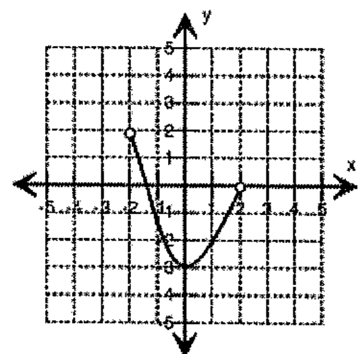


Domian: $\{11,13,14\}$

Range: $\{-4,1,2,6\}$

Function? yes

6.



Domian: $-2 < x < 2$

Range: $3 \leq y < 2$

Function? yes

7. $\{(3,2), (5,7), (1,4), (9,2), (3,7)\}$

Domian: $\{1,3,5,9\}$

Range: $\{2,4,7\}$

Function? no

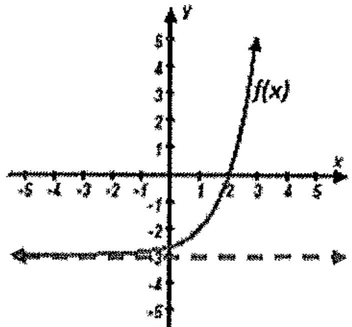
8. $\{(6,2), (3,5), (9,0), (5,7), (8,1)\}$

Domian: $\{3,5,6,8,9\}$

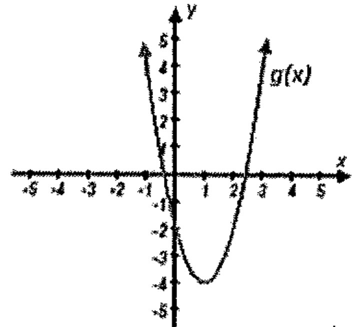
Range: $\{0,1,2,5,7\}$

Function? yes

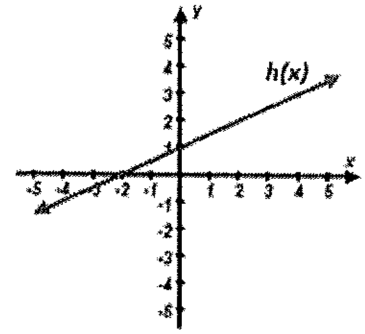
Determine the Domain and Range of each of the following graphed functions (using Interval and Set Notations).



9.
11.



10.



Domain (INTERVAL): $(-\infty, \infty)$

Domain (SET): $-\infty < x < \infty$

Range (INTERVAL): $(-3, \infty)$

Range (SET): $-3 < y < \infty$

Domain (INTERVAL): $(-\infty, \infty)$

Domain (SET): $-\infty < x < \infty$

Range (INTERVAL): $[-4, \infty)$

Range (SET): $-4 \le y < \infty$

Domain (INTERVAL): $(-\infty, \infty)$

Domain (SET): $-\infty < x < \infty$

Range (INTERVAL): $(-\infty, \infty)$

Range (SET): $-\infty < y < \infty$

12. What values of x would make this relation not be a function?

(3,2); (x,4); (5,7); (8,-1)　　3, 5, 8

13. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of [−∞, ∞)** ?

linear

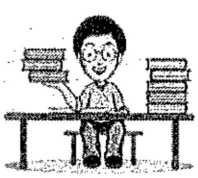
14. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of (2, ∞)**?

Exponential

15. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of [− 5, ∞)**?

Quadratic

16. An author is selling autographed copies of his book at a stand in a bookstore in the mall and charging $12 per copy. The author brought a total of 40 books with him to sell at his stand. If the function $p(x) = 12x$ represents the gross profit the author could make during the time he is sitting at the stand, determine the appropriate domain and range.



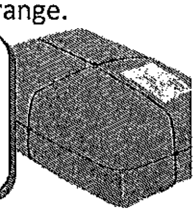
Domain: $0 \le x \le 40$

Range: $0 \le y \le 480$

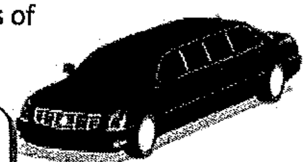
17. A postal company delivers packages based on their weight but will not ship anything over 50 pounds. The company charges \$0.50 per pound to deliver the package anywhere in the United States. If we consider this situation a function where the number of pounds, ***x***, is the independent variable and the cost in dollars, ***y***, is the dependent variable determine the domain and range.

Domain: $0 < x \leq 50$

Range: $0 < y \leq 25$

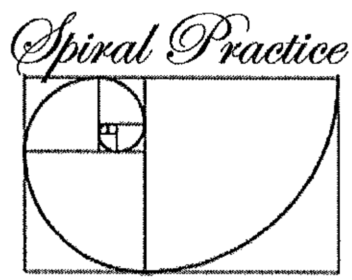


18. A limousine company rents their limousine by the hour. The company charges \$85 per hour. The minimum time is 2 hours and a maximum of 12 hours. If we consider this situation a function where the number of hours, ***x***, is the independent variable and the cost in dollars of renting the limousine, ***y***, is the dependent variable determine the domain and range.



Domain: $2 \leq x \leq 12$

Range: $170 \leq y \leq 1020$



19. Annita makes cupcakes to sell at a bake sale. She is fills each box with 4 cupcakes. How many boxes will Annita need to hold 104 cupcakes?

26 boxes

20. $f(x) = \frac{4}{7}x + 8$ find the value of x if *f(x)=28*

$x = 35$

21. *f(x)* = 6(4x² – 3x +2) – (3x – 6) find *f(0)*

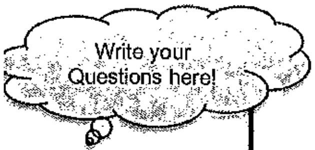
18

22. Put in slope-intercept form and tell the slope and y-intercept

2x – 5y =10

$y = \frac{2}{5}x - 2$

Review your practice and notes to prepare for the mastery check.



4-4: Intercepts

Learning Targets:

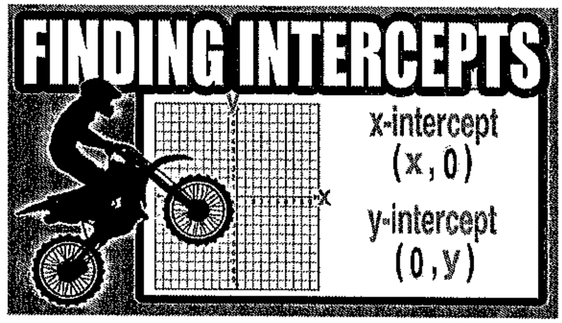
- **I can determine the x and y intercepts of functions.**

F.IF.4

Begin 4-4 Video 1

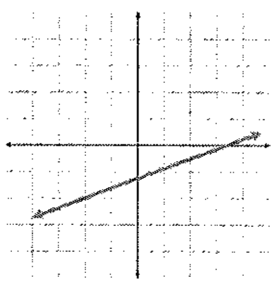
Vocabulary:
- Root
- x-intercept
- y-intercept

The graphical concept of *x*- and *y*-intercepts is pretty simple. The *x*-intercepts are where the graph crosses the __________, and the *y*-intercepts are where the graph crosses the __________.



Example 1:

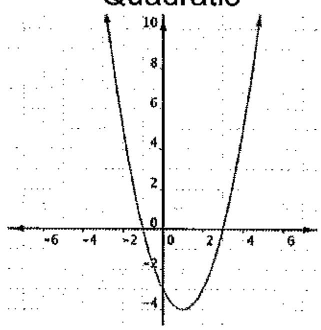
Linear



x-intercept(s):

y-intercept:

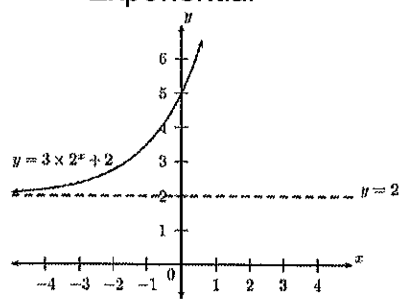
Quadratic



x-intercept(s):

y-intercept:

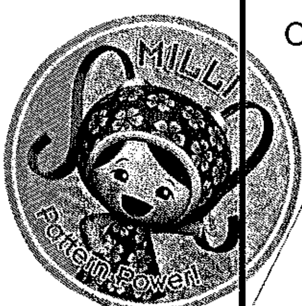
Exponential



x-intercept(s):

y-intercept:

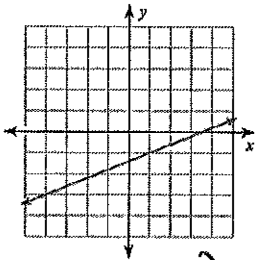
Once again, mathematics offers some patterns that we should take note of.



How many can I expect to see?			
	Linear	**Quadratic**	**Exponential**
x-intercept	One	Zero, One, Two	Zero, One
y-intercept	One	One	One

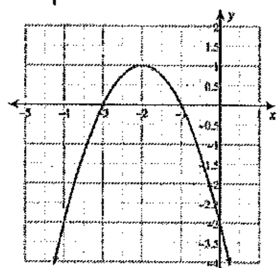
You try: Identify the x- and y-intercepts in the functions below.

1.



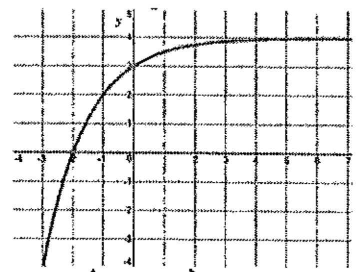
x-int(s): $(3.5, 0)$
y-int: $(0, -1.5)$

2.



x-int(s): $(-3,0)(-1,0)$
y-int: $(0,-3)$

3.

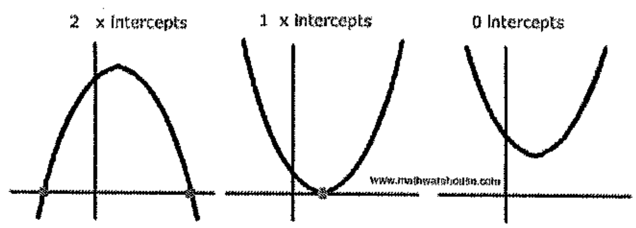


x-int(s): $(-2,0)$
y-int: $(0,3)$

Begin 4-4 Video 2

Finding intercepts from a graph is very straightforward. The problems start when we try to deal with intercepts algebraically.

x-intercept:
The x-coordinate of the point at which the graph of an equation crosses the x-axis, where ________.

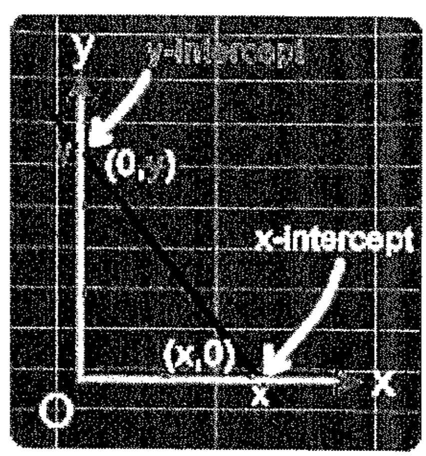


Steps to find the x-intercept:
1. Plug in $y = 0$.
2. Always write as an ordered pair. ______

Example 2: Find the x-intercept of the given equation.
$y = 5x + 4$

Note: At this point, you are only prepared for finding x-intercepts of Linear functions algebraically. Quadratic and Exponential x-intercepts can be found graphically now. Algebraic methods will be covered in later units.

Example 3: Find the x-intercept of the given equation.
$-5x + y = 4$



You try: Find the x-intercept of the given equations.

1. $y = -2x + 4$

$(2,0)$

2. $4x + 3y = 4$

$(1,0)$

Write your Questions here!

Begin 4-4 Video 3

Remember

In a Linear Equation, the y-intercept can be found by putting the function in slope-intercept form and finding the "b".
$y = mx + b$

y-intercept:
The y-coordinate of the point at which the graph of an equation crosses the y-axis, where ___________.

Steps to find the y-intercept:
1. Plug in $x = 0$.
2. Always right as an ordered pair. _________

Example 4: Find the y-intercept of the given equation.
$y = 5x + 4$

Example 5: Find the y-intercept of the given equation.
$3x^2 - 5x + 2 = y$

Example 6: Find the y-intercept of the given equations.
$y = -2(2)^x + 4$

You try: Find the y-intercept of the given equations.

1. $y = -2x + 4$

(0,4)

2. $y = 4x^2 + 3x - 4$

(0,-4)

3. $3^x - 4 = y$

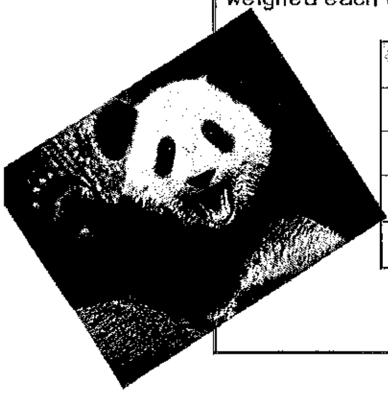
(0,-3)

Begin 4-4 Video 4

Interpreting the intercepts.

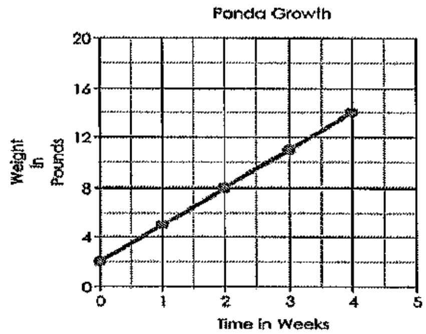
Example 7: Zoo Atlanta recently celebrated the birth of two new baby pandas!

Mochi the panda cub has been measured and weighed each week since she was born.



Weeks	Weight
0	1
1	5
2	9
3	13

Mochi's brother is **Kappa**. His weight has been charted on the graph below.



What is the y-intercept for Mochi and for Kappa? What is the significance of these values for the baby pandas?

Neither panda has an x-intercept. Why? What would that mean in the context of this problem?

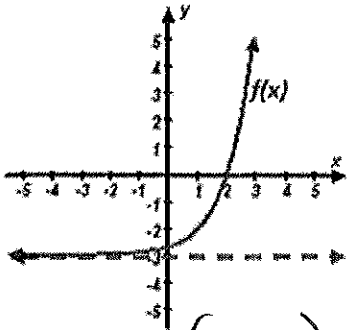
Write your Questions here!

Practice Makes Better

Practice 4 – 4: Intercepts

Find the x- and y-intercepts for each function.

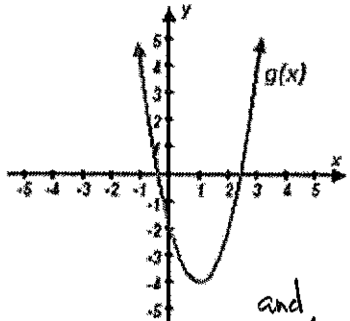
1.



x-int(s): (2,0)

y-int: (0, −2.75)

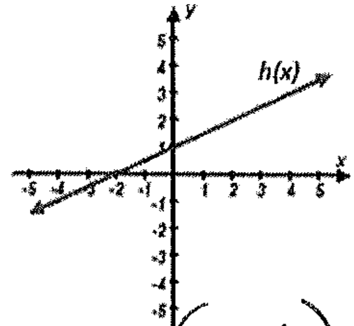
2.



x-int(s): (−.5,0) and (2.5,0)

y-int: (0,−2)

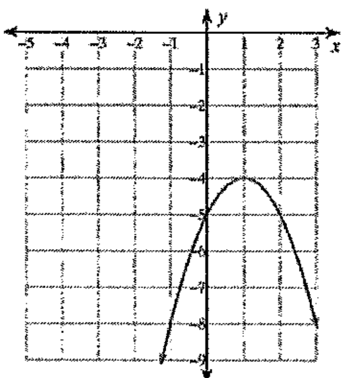
3.



x-int(s): (−2,0)

y-int: (0,1)

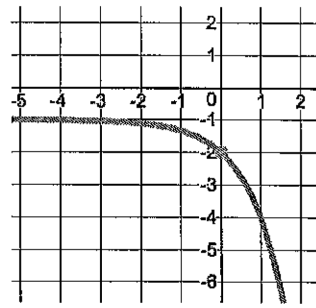
4.



x-int(s): none

y-int: (0,−5)

5.



x-int(s): none

y-int: (0,−2)

Find the x-intercept of the following functions.

6. $3x + 7y = 12$

(4,0)

7. $y = \frac{1}{2}x - 7$

(14,0)

8. $y = -5x + 3$

$(\frac{3}{5},0)$

9. $y = 0.2x + 6$

(−30,0)

10. $y = 4x + 3$

(−0.75,0)

11. $2x - y = -10$

(−5,0)

Find the y-intercept of the following functions.

12. $3x + 7y = 12$

$(0,\frac{12}{7})$

13. $y = 4^{x+3}$

(0,64)

14. $y = -2(3^x) + 7$

(0,5)

15. $y = -2(x-4)^2 + 6$

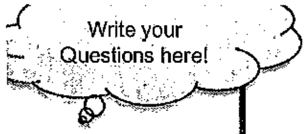
(0,−26)

16. $y = 4x + 3$

(0,3)

17. $y = 12x^2 - 5x + 3$

(0,3)



18. Two contestants on The Biggest Loser are Valerie and Oscar. Their weight loss progress is shown below.

Valerie's weight loss is shown by this function, where W is her weight in pounds and t is the time in weeks.	Oscar's weight loss is tracked in the table below.
$W = 235 - 2.5t$	

Weeks	0	2	5	6
Weight	247	243	237	235

What is the y-intercept for Valerie and for Oscar? What is the significance of these values for the contestants? Who is losing weight faster?

What mathematical term would we use to describe the amount of weight they are loosing each week?

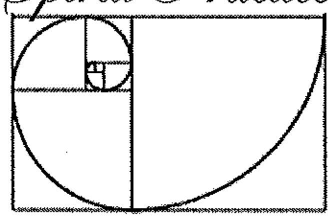
a.) V- 235
Oscar- 247

b) starting weight

c) Oscar

d) slope or average rate of Change

Spiral Practice



Add or subtract the Polynomials

19. $(4a^2 - 2a + 1) - (a^3 - 2a + 3)$

$-a^3 + 4a^2 - 4a + 4$

20. $(3x^3 + 4x + 14) + (-4x^2 + 21)$

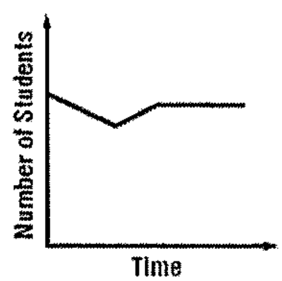
$3x^2 - 4x^2 + 4x + 35$

21. $(x^2 + 11xy - 3y^2) + (-2x^2 - xy + 4y^2)$

$-x^2 + 10xy - y^2$

22. The graph below shows the number of students enrolled at Edison Junior High School. Describe the change in the number of students over time.

attendence dropped, then rose, then stayed the same



Review your practice and notes to prepare for the mastery check.

Write your Questions here!

4-5: Intervals of Positive & Negative
Intervals of Increasing & Decreasing

Learning Targets:

- **I can identify the intervals of increase & decrease of a function.**
- **I can identify intervals of positive & negative of a function.**

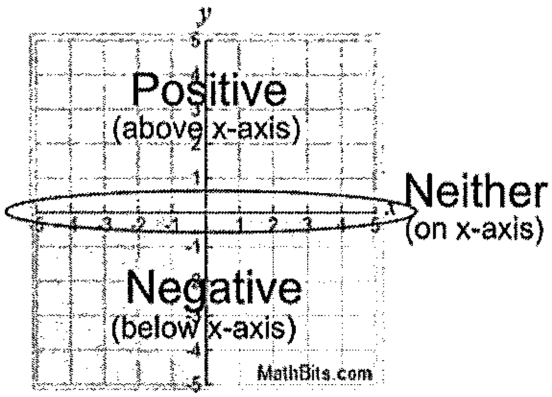
F.IF.4

Begin 4-5 Video 1

Positive/Negative

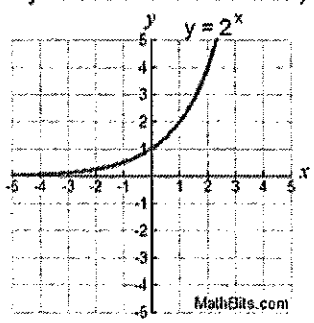
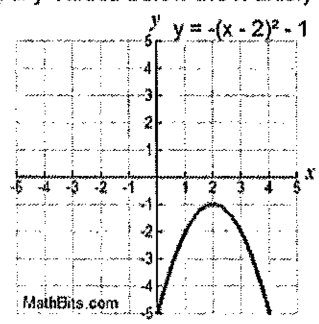
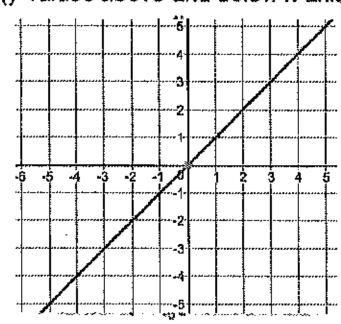
Not Zero!

y-values positive or y-values negative



- The positive regions of a function are those intervals where the function is ________ the x-axis. It is where the ____________ are positive (not zero).
- The negative regions of a function are those intervals where the function is ________ the x-axis. It is where the y-values are ____________ (not zero).
- y-values that are on the x-axis are neither positive nor negative. The x-axis is where ____________.

Example 1:

Some functions are positive over their entire domain (All y-values above the x-axis.)	Some functions are negative over their entire domain. (All y-values below the x-axis.)	Some functions have both positive and negative regions. (y-values above and below x-axis)
		
positive: $-\infty < x < +\infty$ or "all Reals", or $(-\infty,+\infty)$	negative: $-\infty < x < +\infty$ or "all Reals", or $(-\infty,+\infty)$	positive: $x > 0$ or $(0,+\infty)$ negative: $x < 0$ or $(-\infty,0)$ (do not include zero)

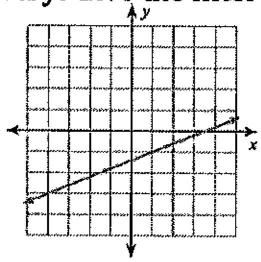
Revisit 3-3 Video 2 if you need a refresher!

Secret to Finding the Intervals!

The secret to correctly stating the intervals where a function is positive or negative is to remember that the intervals **ALWAYS** pertain to the locations of the x-values. Think of reading the graph from left to right along the x-axis. Do NOT read numbers off the y-axis for the intervals. **Stay on the x-axis!**

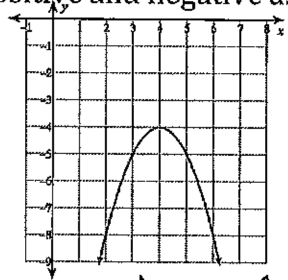
You try: Give the intervals of positive and negative as applicable.

1.



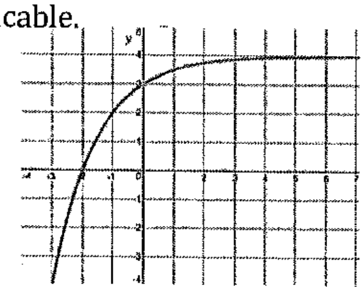
Postive $x > 3.5$
neg $x < 3.5$

2.



always negative

3.



Positive $x > -2$
neg $x < -2$

Begin 4-5 Video 2

Increasing/Decreasing

When looking for sections of a graph that are increasing or decreasing, be sure to look at (or "read") the graph **from left to right.**

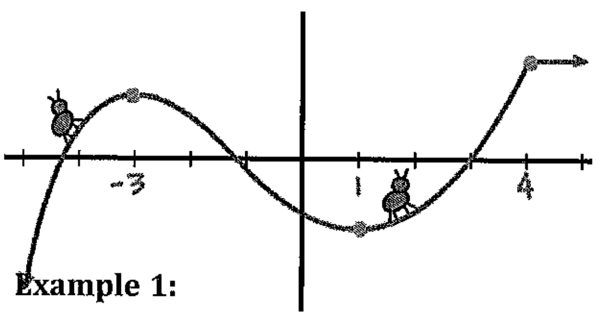
Increasing: A function is *increasing*, if as x increases (reading from left to right), y also _______________.

- In plain English, as you look at the graph, from left to right, the graph goes _________.
- The graph has a __________________ *slope*.

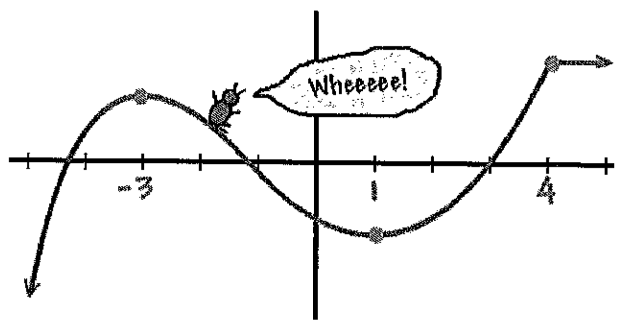
Decreasing: A function is *decreasing*, if as x increases (reading from left to right), y decreases.

- In plain English, as you look at the graph, from left to right, the graph goes down-hill.
- The graph has a *negative slope*.

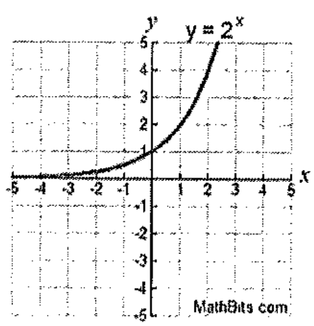
If Pierre is climbing uphill, then the graph is increasing:

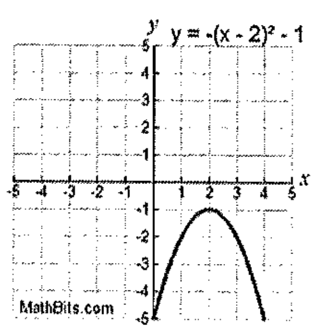


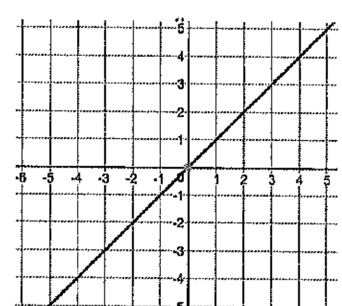
If Pierre is going downhill, then the graph is decreasing:



Example 1:





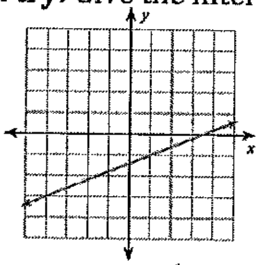


Write your Questions here!

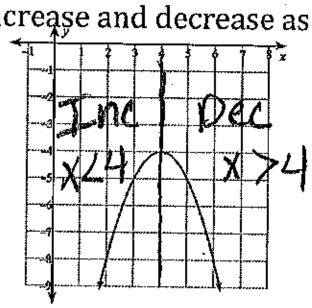
Intervals of increasing and decreasing
ALWAYS pertain to x-values.
Do NOT read numbers off the y-axis.
Stay on the x-axis for these intervals!

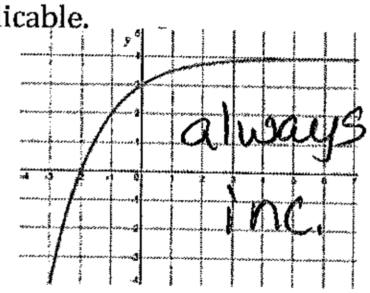
Function Type	What Can you expect?
Linear	Will either increase or decrease for $\mathbb{R}$ depending on the slope. • Increasing for positive slope. • Decreasing for negative slope.
Quadratic	Will have a section of increase AND a section of decrease
Exponential	Will either increase or decrease for $\mathbb{R}$ depending on whether it is a growth or decay problem. • Increasing for growth. • Decreasing for decay.

You try: Give the intervals of increase and decrease as applicable.

1.


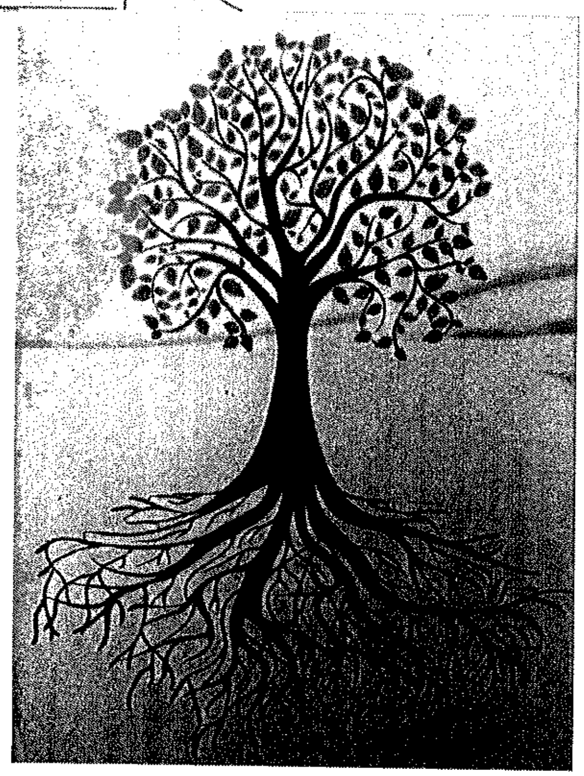
always inc.

2.


3.


Begin 4-5 Video 3

The Tree Metaphor.
It can get confusing when positive, negative, increasing and decreasing are all together. How to keep it straight? Think of a tree.



The parts of the tree that are __________ ground (x-axis) are **positive**.

The parts of the tree that are __________ ground are **negative**.

Branches growing up towards the _________? The height is **increasing**.

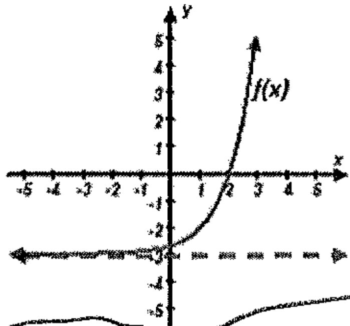
Branches turning back towards the _________? The height is **decreasing.**

Write your Questions here!

Practice Makes Better

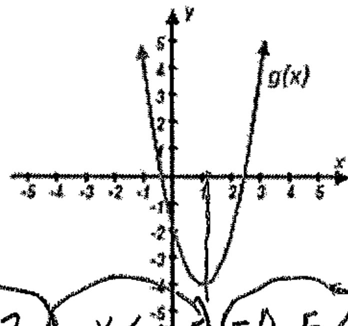
Practice 4 – 5: Intervals of Positive & Negative/Increasing & Decreasing
Find the intervals over which each function is positive and negative. Find the intervals over which each function is increasing and decreasing.

1.



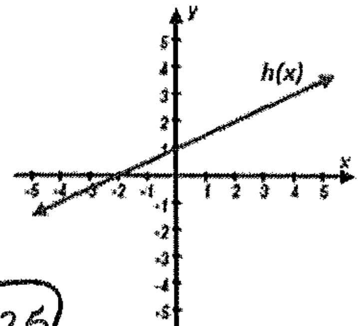
Positive: $x > 2$ Negative: $x < 2$
Increase: always Decrease: never

2.



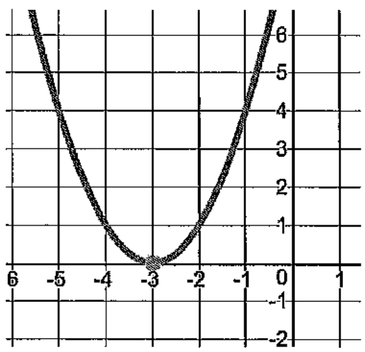
Positive: $x < 0.5$ and $x > 2.5$ Negative: $-0.5 < x < 2.5$
Increase: $x > 1$ Decrease: $x < 1$

3.



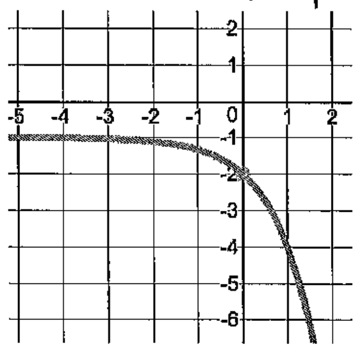
Positive: $x > -2$ Negative: $x < -2$
Increase: always Decrease: never

4.



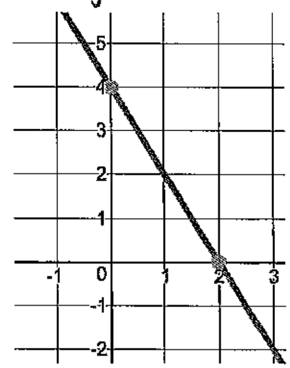
Positive: $x \neq 3$ Negative: never
Increase: $x > -3$ Decrease: $x < -3$

5.



Positive: never Negative: always
Increase: never Decrease: always

6.



Positive: $x < 2$ Negative: $x > 2$
Increase: never Decrease: always

Spiral Practice

This WILL be on your mastery check!

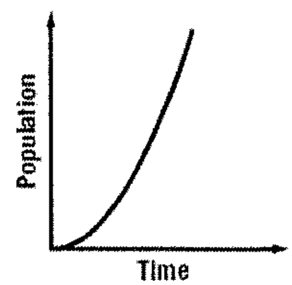
Change the following fractions into decimals, then tell whether the decimal is terminating or repeating

7. $\frac{-13}{15}$ Repeating $-0.8\overline{6}$

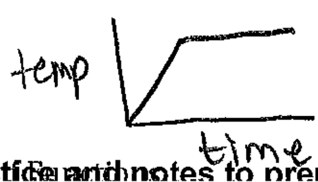
8. $-9\frac{5}{8}$ terminating -9.625

9. The graph below shows the population of bacteria in a dish. Describe the change in population over time.

increasing exponentially



10.. An oven is being preheated in order to bake a cake. Sketch a qualitative graph to represent the temperature of the oven over time.



Write your Questions here!

4-6: Special Characteristics

Learning Targets:

- **I can do math.**

F.IF.4

Vocabulary:
- Asymptote
- Axis of Symmetry
- Vertex

Begin 4-6 Video 1

Special Features of Quadratic Functions

VERTEX

The minimum or maximum point of a quadratic function

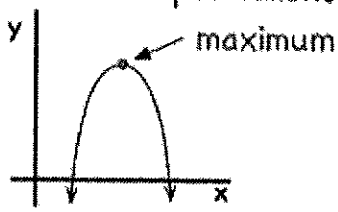
The vertex should always be written as an ordered pair.

A parabola that opens upward contains a vertex that is a ________________ point.

A parabola that opens downward contains a vertex that is a ________________ point.

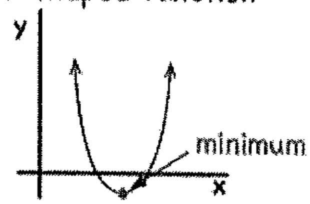
MAXIMUM OF A QUADRATIC

Highest point on the graph of an upside-down U shaped function



MINIMUM OF A QUADRATIC

Lowest point on the graph of a U-shaped function



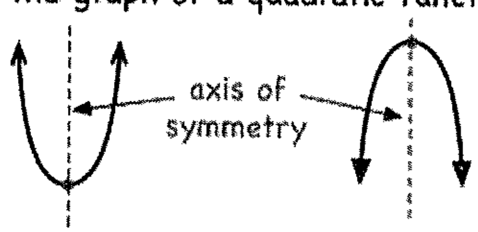
If the vertex is a *minimum*, then the range is *all real numbers* ____________ *than or equal to the y-value*.

If the vertex is a *maximum*, then the range is *all real numbers* _________ *than or equal to the y-value*.

We sometimes refer to the maximum and minimum as the extrema.

AXIS OF SYMMETRY

The vertical line through the vertex on the graph of a quadratic function

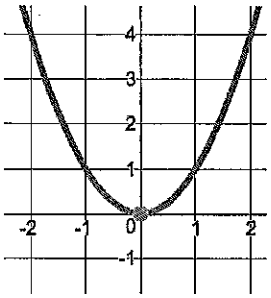


An **axis of symmetry** (also known as a **line of symmetry**) will divide the parabola into ____________. The line of symmetry is always a ____________ line of the form _________, where *n* is a real number. The axis of symmetry is often abbreviated as AOS.

Write your Questions here!

Example 1: Identify the vertex of each function. Is the vertex a minimum or maximum? Write an equation for the axis of symmetry.

a.

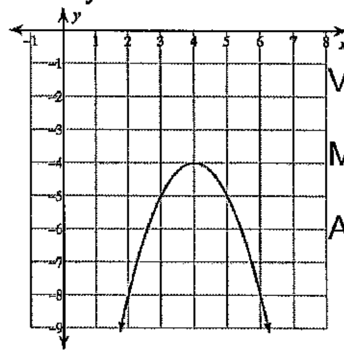


Vertex:

Min or Max?

Axis of Symmetry:

b.



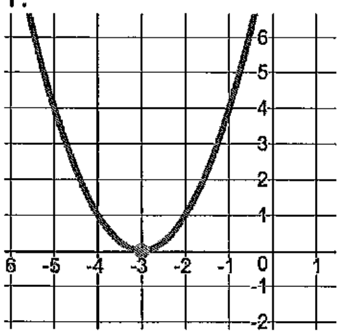
Vertex:

Min or Max?

Axis of Symmetry:

You try:

1.

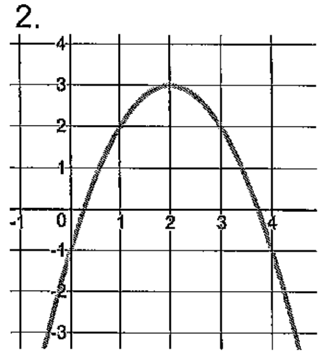


Vertex: (-3,0)

Min or Max? min 0

Axis of Symmetry: x = -3

2.



Vertex: (2,3)

Min or Max? max 3

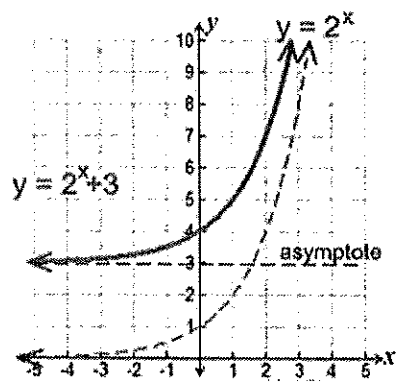
Axis of Symmetry: x = 2

Begin 4-6 Video 2

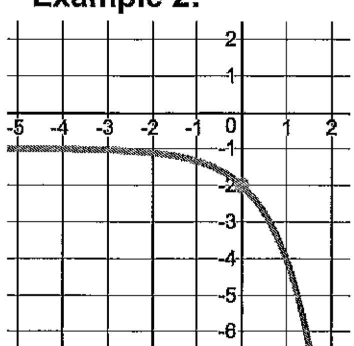
Special Features of Exponential Functions

An asymptote is a line that a curve ________________ (but does not touch) as it heads away from the origin.

- Every exponential function has a horizontal asymptote.
- Functions of the form $y = a(b)^x$ (the exponential parent function) have horizontal asymptotes at ________ (the x-axis).
- Vertical shifts will shift the asymptote along with all the other points.
- Horizontal asymptotes should be written as an equation of the form ________, where c is the value of the vertical translation.

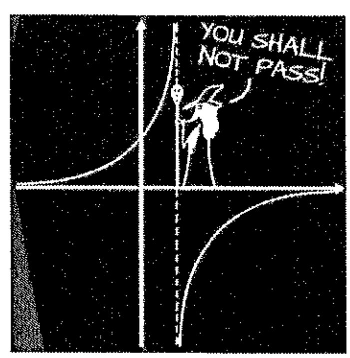


Example 2:



Example 3:

$y = 3^x + 4$



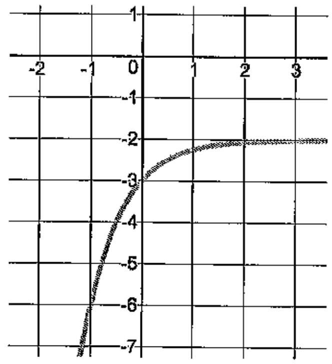
Write your Questions here!

You try: Write the equation of the horizontal asymptote for the following functions.

1. $y = 2^x + 1$

$y = 1$

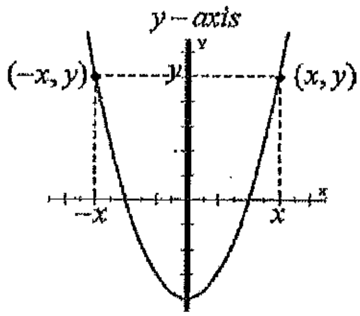
2.



$y = -2$

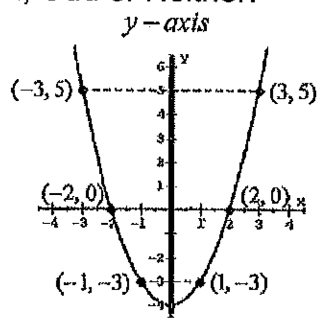
Begin 4-6 Video 3

Functions can be classified as Even, Odd or Neither.

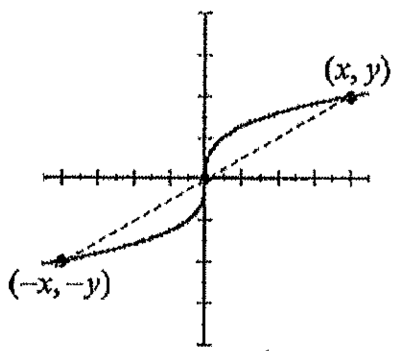


$f(-x) = f(x)$

Even Functions have symmetry about the <u>y-axis</u>.

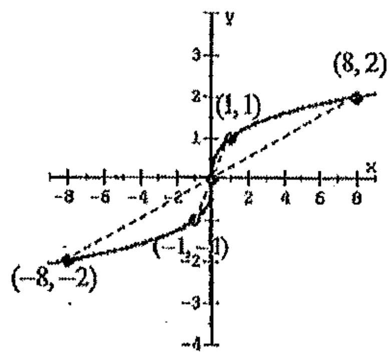


$y = x^2 - 4$



$f(-x) = -f(x)$

Odd Functions have symmetry about the <u>origin</u>.



$y = \sqrt[3]{x}$

Write your Questions here!

Hint: If you are getting lost in the x's, try plugging in 2 and −2 to compare results.

Test to determine whether a function is Even, Odd or Neither:

Replace x with –x and compare the result to the original function.

- If $f(-x) = f(x)$, the function is __________.
- If $f(-x) = -f(x)$, the function is __________.
- If $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is __________.

Type	What Can you expect?
Linear	• Sometimes Even ○ Only horizontal lines ○ Of the form, $f(x) = b$ • Sometimes Odd ○ Only lines that go through the origin ○ Of the form $f(x) = mx$ • Sometimes Neither
Quadratic	• Sometimes Even • Sometimes Neither
Exponential	• Always Neither!

Example 4:
$f(x) = -3x^2$

Example 5:
$f(x) = -3x$

Example 6:
$f(x) = -3(2)^x$

You try:

1. $f(x) = (3)^x$

neither

2. $f(x) = x^2 + x + 6$

neither

3. $f(x) = x + 6$

neither

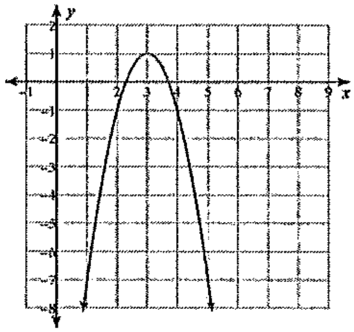
Practice Makes Better

Write your Questions here!

Practice 4 – 6: Special Characteristics

Identify the vertex of each function. Is the vertex a minimum or maximum? Write an equation for the axis of symmetry.

1) $y = -2x^2 + 12x - 17$



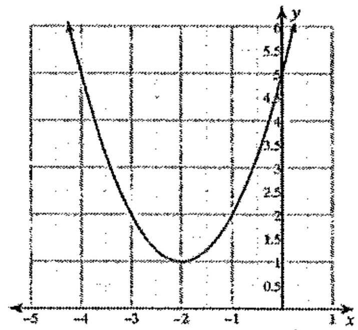
Vertex: (3, 1)

Min or Max? max 1

Axis of Symmetry:

x = 3

2) $y = x^2 + 4x + 5$



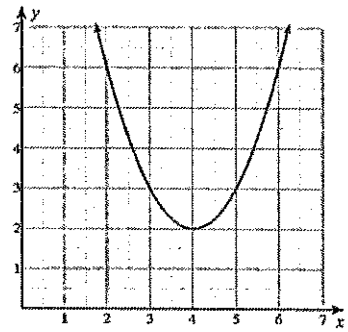
Vertex: (-2, 1)

Min or Max? min 1

Axis of Symmetry:

x = -2

3) $y = x^2 - 8x + 18$



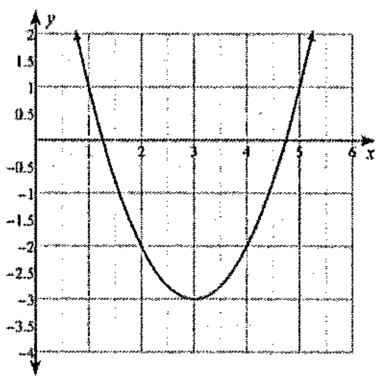
Vertex: (4, 2)

Min or Max? min 2

Axis of Symmetry:

x = 4

4) $y = x^2 - 6x + 6$



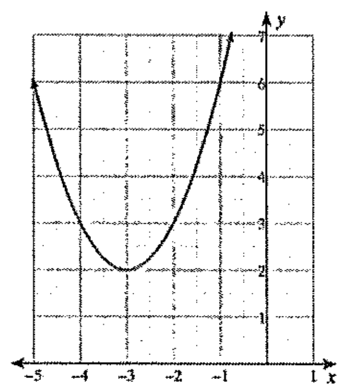
Vertex: (3, -3)

Min or Max? min -3

Axis of Symmetry:

x = 3

5) $y = x^2 + 6x + 11$



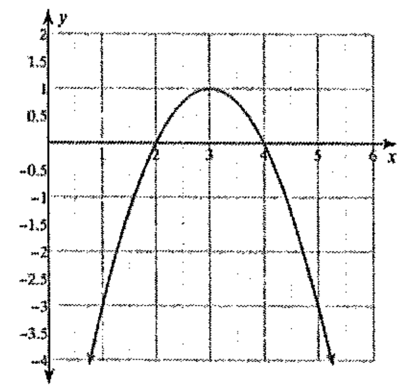
Vertex: (-3, 2)

Min or Max? min 2

Axis of Symmetry:

x = -3

6) $y = -x^2 + 6x - 8$



Vertex: (3, 1)

Min or Max? max 1

Axis of Symmetry:

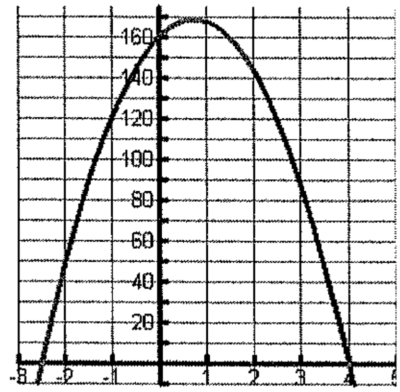
x = 3

Write your Questions here!

Wil E. Coyote is catapulting a boulder off a cliff to hit the road runner. Let t represent the number of seconds that the boulder catapults off the cliff and h(t) denote the height of the boulder, in feet, above the base of the cliff. Ignoring air resistance, we can use the following formula to express the path of the boulder: $h(t) = -16t^2 + 24t + 160$

7. What does the x axis represent? time The y axis? height above ground

8. What part of the graph is insignificant? Why?
negative amounts
can't have negative time or height



9. What was the height of the boulder before it was launched? What special point on the graph is associated with this information? 160 ft, y-intercept

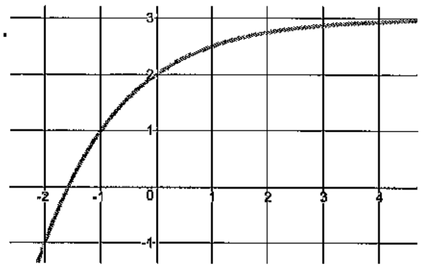
10. How long will it take before the boulder reaches the bottom of the cliff? What special point on the graph is associated with this information?
4 sec., x-int

11. After how many seconds does the boulder change direction? ≈0.75 secs
How high is the boulder when it changes direction? 170 ft
What is this significant point called on the graph? vertex

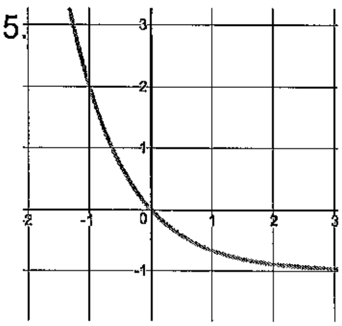
12. How high above the starting point does the boulder begin to change direction?
10 ft

13. If Wil E. Coyote changes his mind, how many seconds does he have to stop the boulder from going over the cliff? ≈1.7 sec.

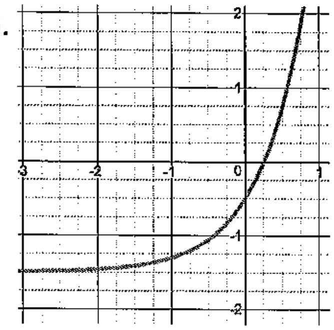
Write the equation of the horizontal asymptote for the following functions.

14.


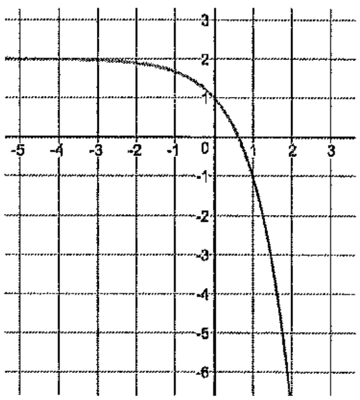
$y=3$

15.


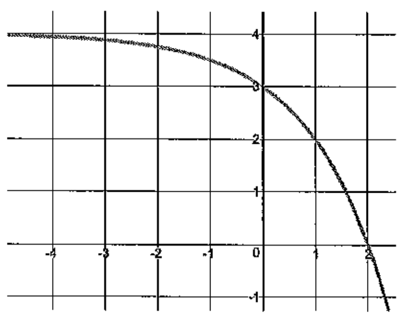
$y=-1$

16.


$y=-1.5$

17.


$y=2$

18.


$y=4$

19. $y = 3^x + 9$

$y=9$

20. $f(x) = 3^x$

$y=0$

21. $y = 3^x - 2.4$

$y=-2.4$

Determine whether each of the following functions are even, odd or neither.

22. $f(x) = 3^x$

Neither

23. $y = 2x^2 + 3x - 9$

Neither

24. $y = 2x + 9$

Neither

25. $f(x) = -(\frac{1}{3})^x$

Neither

26. $y = 4x$

odd

27. $y = x^2 + 9$

even

Spiral Practice

Write your Questions here!

This WILL be on your mastery check!

Convert the decimals into fractions, make sure to reduce fractions to simplest form when possible and write improper fractions as mixed numbers

28. 0.4 $2/5$

29. 2.093 $2\frac{93}{1000}$

30. 0.324 $\frac{81}{250}$

31. A football club is hiring a painter to paint a mural on the concession stand wall. The painter charges an initial fee plus $25 an hour. After 12 hours of work, the football club owed $350. Create a function to represent the situation. Find and interpret the rate of change and initial value.

RoC $25/hr
initial amt = $50

32. After writing part of his novel, Thomas is now writing consistent number of pages each week. The table below tracks Thomas' progress. Create a function to represent the situation. Find and interpret the rate of change.

Number of weeks	4	6	9
Pages Written	85	117	165

RoC 16 pages/week

Review your practice and notes to prepare for the mastery check.

Write your Questions here!

4-7: End Behavior

Learning Targets:

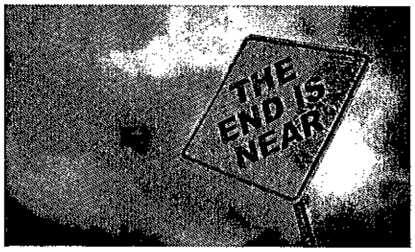
- **I can identify the end behavior of a function.**

F.IF.4

Begin 4-7 Video 1

Vocabulary:
- End Behaviors

One last characteristic that we can use to describe a function is end behavior. End behavior describes what happens to the ____________ as the __________ approaches infinity or negative infinity.



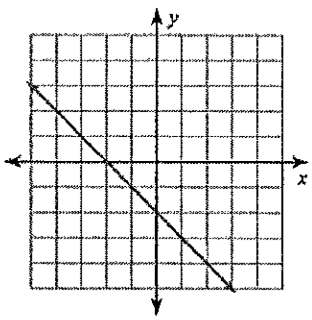
There are a few different ways to ask about end behavior:

As x$\rightarrow -\infty, y\rightarrow$ ____________ (Read as x approaches negative infinity, y approaches____?) As x$\rightarrow \infty, y\rightarrow$ ____________ (Read as x approaches positive infinity, y approaches ____?)
As x increases , y __________ As x decreases , y _________
As x increases , $f(x)$ __________ As x decreases , $f(x)$ _________

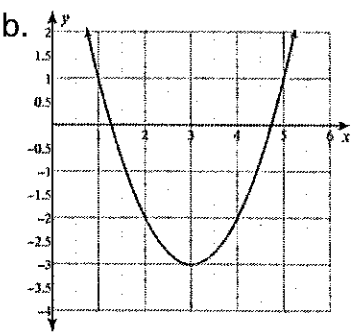
Function Type	What Can you expect?
Linear	• One side will approach ∞. • One side will approach $-\infty$.
Quadratic	• Both sides will approach the same thing.
Exponential	• One side will approach ∞ or $-\infty$. • One side will approach the asymptote.

Example 1: Describe the end behavior of the functions below.

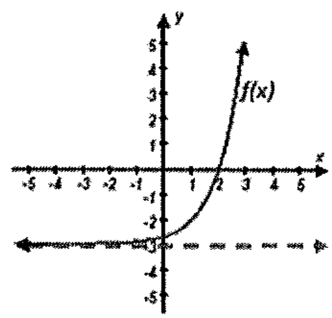
a.



b.



c.



As x$\rightarrow -\infty, y\rightarrow$ ____________

As x$\rightarrow \infty, y\rightarrow$ ____________

As x increases, y ___________

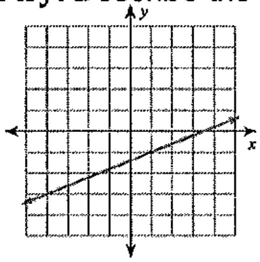
As x decreases, y ___________

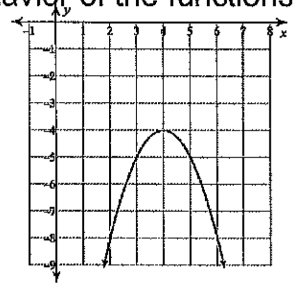
As x decreases, f(x) ___________

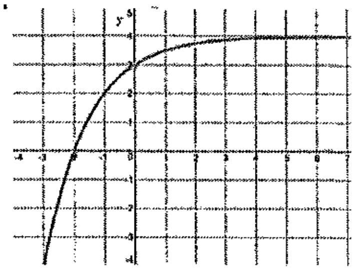
As x increases, f(x) ___________

Write your Questions here!

You try: Describe the end behavior of the functions below.

1. 

2. 

3. 

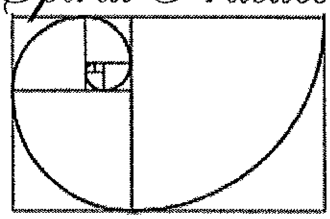
1.	2.	3.
As $x \to -\infty, y \to$ $-\infty$	As x increases, y decreases	As x decreases, f(x) decreases
As $x \to \infty, y \to$ ∞	As x decreases, y decreases	As x increases, f(x) approaches 4

Practice Makes Better

Practice 4 – 7: End Behavior

Surprise! We are going to do spirals first this time.

Spiral Practice



This WILL be on your mastery check!

1. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 3 and 4. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raymond's Solution:

$\mathbf{a^2 + b^2 = c^2}$

$\mathbf{4^2 + 3^2 = c^2}$

$\mathbf{16 + 9 = c^2}$ ← needs to √ both sides

$\mathbf{25 = c}$

Correct Solution:

2. Evaluate the expressions when a = 156 and b = 12

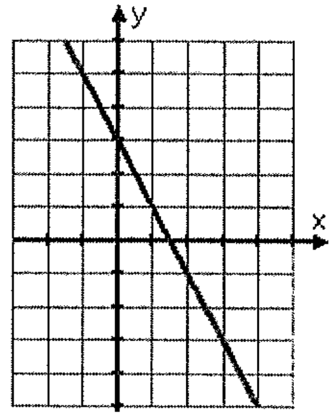
   $\sqrt{a-b}$

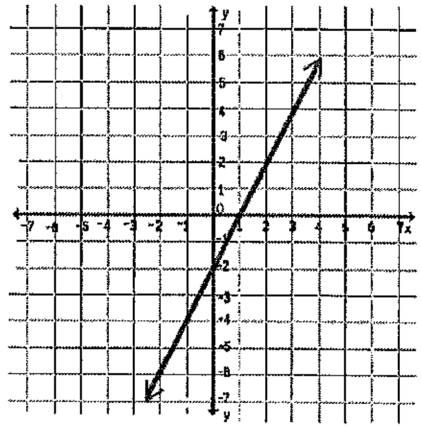
   12

3. Marcus told his mother that the area of the square office at his school is 125 ft^2. His mom asked him for the length of one side of the office. What is the length of the side of the office?

   $5\sqrt{5} \approx 11.18$

Summarize the characteristics of the following functions.





a. rate of change (slope)

-2

b. domain

$\mathbb{R}$

c. range

$\mathbb{R}$

d. x-intercept

(1.5,0)

e. y-intercept

(0,3)

f. interval of inc.

Never

g. interval of dec.

always

h. interval of pos.

$x < 1.5$

i. interval of neg.

$x > 1.5$

j. End Behavior:

as $x \to -\infty, y \to$ ∞

as $x \to \infty,\ y \to$ $-\infty$

a. rate of change (slope)

2

b. domain

$\mathbb{R}$

c. range

$\mathbb{R}$

d. x-intercept

(1,0)

e. y-intercept

(0,−2)

f. interval of inc.

Always

g. interval of dec.

Never

h. interval of pos.

$x > 1$

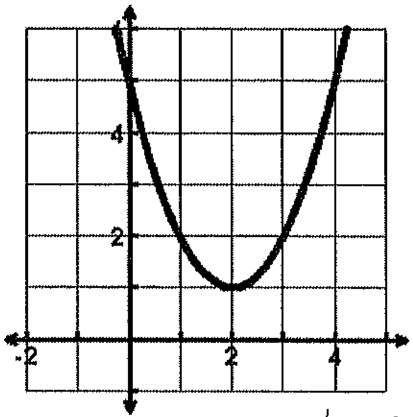
i. interval of neg.

$x < 1$

j. End Behavior:

as $x \to -\infty, y \to$ $-\infty$

as $x \to \infty,\ y \to$ ∞



a. Axis of Symmetry: X=2

b. Vertex: (2,-1)

Max: N/A or Min: -1

c. x-intercept

none

d. Y-intercept: (0,5)

e. Domain: ℝ

f. Range: y > 2

g. Interval of Increasing: x > 2

h. Interval of Decreasing: x < 2

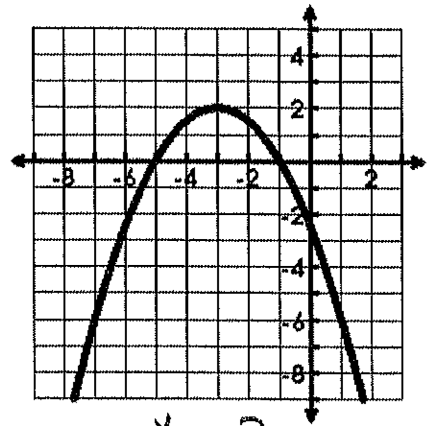
i. Interval of Positive: always

j. Interval of Negative: never

k. Rate of Change over [1,3]: 0

l. End Behavior:

As $x \to -\infty, y \to$ ∞
As $x \to \infty, y \to$ ∞



a. AOS: X=-3

b. Vertex: (-3,2)

Max: 2 or Min: N/A

c. x-intercept(s): (-5,0), (-1,0)

d. Y-intercept: (0,-2)

e. Domain: ℝ

f. Range: y < 2

g. Interval of Increasing: x < -3

h. Interval of Decreasing: x > -3

i. Interval of Positive: -5 < x < -1

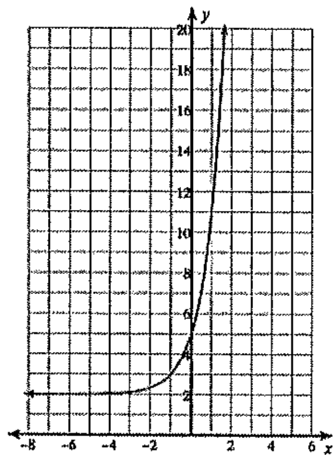
j. Interval of Negative: x < -5 and x > -1

k. Rate of Change over [-7,0]: 6/7

l. End Behavior:

As $x \to -\infty, y \to$ -∞
As $x \to \infty, y \to$ ∞

Write your Questions here!



a. Asymptote: y = 2

b. X-intercept: none

c. Y-intercept: (0,5)

d. Domain: ℝ

e. Range: y > 2

f. Interval of Increasing: always

g. Interval of Decreasing: never

h. Interval of Positive: always

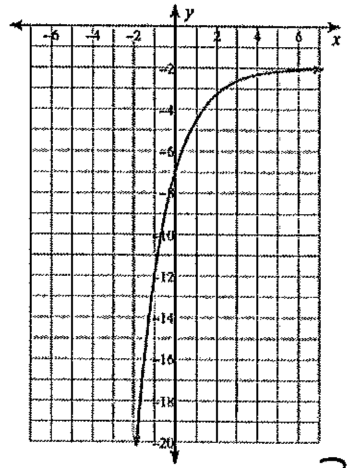
i. Interval of Negative: never

j. Rate of Change over [-1,0]: 2

k. End Behavior:

$As\ x \to -\infty, y \to$ 2

$As\ x \to \infty, y \to$ ∞



a. Asymptote: y = -2

b. X-intercept: none

c. Y-intercept: (0,-7)

d. Domain: ℝ

e. Range: y < -2

f. Interval of Increasing: always

g. Interval of Decreasing: never

h. Interval of Positive: never

i. Interval of Negative: always

j. Rate of Change over [-1,0]: 5

k. End Behavior:

$As\ x \to -\infty, y \to$ $-\infty$

$As\ x \to \infty, y \to$ -2

Review your practice and notes to prepare for the mastery check.

Unit 4 Study Guide

For the questions 1-3, tell whether the following scenarios describe a linear, exponential, or quadratic function.

1. Given a function exhibits the following end behavior:
As x decreases, f(x) increases
As x increases, f(x) increases
quadratic

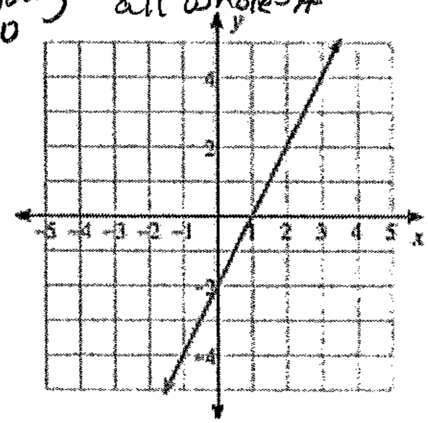
2. Given a function exhibits the following end behavior:
As x decreases, f(x) increases
As x increases, f(x) approaches 4
exponential

3. Given a function exhibits the following end behavior:
As x decreases, f(x) increases
As x increases, f(x) decrease
linear

4. A machine makes bolts at an average of 250 per hour for up to 10 hours. Let y be the number of bolts that the machine can make for a given x amount of time. What is the domain of the function? What is the range of the function?
D: $\{0 \le x \le 10\}$ R: $\{0 \le x \le 2500\}$ * technically only whole #

5 .Marco opened a savings account with \$70. Each month, he plans to save \$40. Write a function that represents this situation. What is the domain of the function? What does the domain represent?
$y = 40x + 70$ D: $\{1, 2, 3 \ldots\}$ * technically all whole #
months
$x \ge 0$

6-10. Use the graph shown below to answer the following questions.



6. What is the domain of the function?
$\mathbb{R}$

7. What is the range of the function?
$\mathbb{R}$

8. What is the y-intercept of the function?
(0, -2)

9. What is the x intercept of the function?
(1, 0)

10. When x>1, the function is increasing and positive.

11. Find the average rate of change of f(x) over the following intervals.

x	f(x)
-1	17
0	20
1	23
2	26
4	32

A. $-1 \le x \le 2$
3

B. $2 \le x \le 4$
3

What is the asymptote of the exponential functions?

12. $f(x) = -3(2)^x$

$y = 0$

13. $f(x) = 2(1/2)^x + 1$

$y = 1$

14. $f(x) = 4(5)^x - 3$

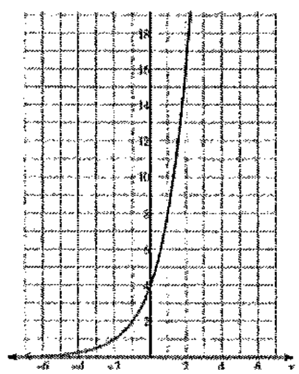
$y = -3$

15. What is the average rate of change for the functions f(x) from x = 1 and x = 3?

$f(x) = -3^x + 2$ -12

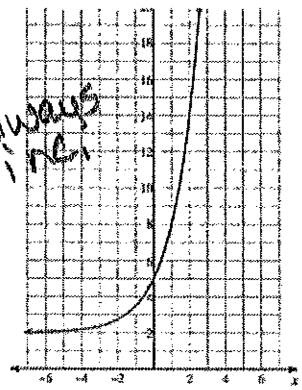
For each of the following functions describe the interval of increase and/or decrease.

16.



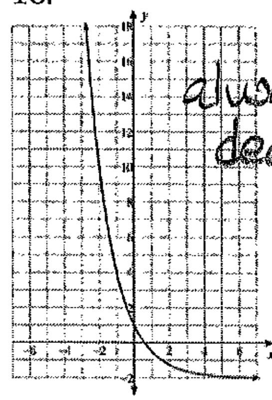
always inc.

17.



always inc.

18.



always decrease

19. For which of the above graph is the following statement true. "as x decreases, f(x) approaches 0"

#16

20. For which of the above graphs is the following statement true. " as x increases, f(x) approaches -2"

#18

21. For which of the above graphs is the following statement true. "as x increases, f(x) approaches infinity"

#17

22. What is the asymptote for 16 $y = 0$ for 17 $y = 2$ and 18 $y = -2$?

23. Which has a greater rate of change from x = 0 to x = 4, f(x) or g(x)?

g(x)

$f(x) = 3^x + 1$

x	g(x)
0	2
1	16
2	25
3	50
4	100

Find the y-intercept. (To find the y-intercept replace x = 0 and solve for y.)

24. $f(x) = 4^x$

(0,1)

25. $f(x) = 2^x - 1$

(0,0)

26. $f(x) = 3(2)^x + 4$

(0,7)

27. $f(x) = 3x - 6$

(0,-6)

28. $f(x) = 4x - 2$

(0,-2)

29. Busy Golf: Rents golf carts for $35 plus $7 per hour.
Cruising Golf rental fees are as follows:

Hours (x)	Total Cost g(x)
2	34
3	44
4	54
5	64
6	74
7	84

Find and compare the y-intercepts of the two functions.

Busy $35 Cruising $14

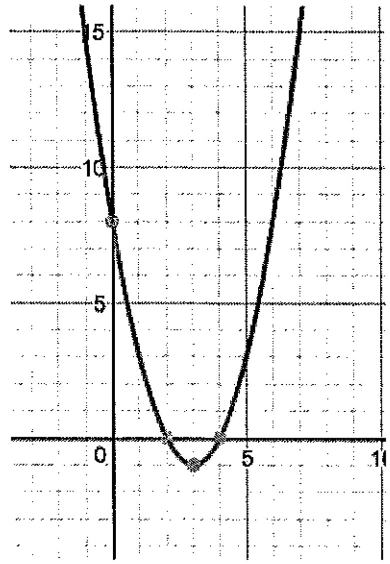
Find and compare the rate for change for each function.

Busy $7 Cruising $10

Based on this information, which rental would you choose? Write a short paragraph explaining why you think Busy Golf or Cruising Golf would be the better choice. Be sure to support your decision with mathematical reasoning.

Cruising under 7 hours
Busy over 7 hours

30.



a. Domain $\mathbb{R}$
b. Range $x > 1$
c. Interval of Increase: $x > 3$
d. Interval of Decrease: $x < 3$
e. What is the end behavior of the graph?
$as\ x \to \infty\ (increases), f(x) \to$ ∞
$As\ x \to -\infty\ (decreases), f(x) \to$ ∞

f. y-intercept: (0,8)
g. x-intercept(s): (2,0) (4,0)

h. What are the coordinates of the vertex?
(3,-1)
i. Give an equation for the Axis of Symmetry.
x=3

j. Is the extreme point a maximum or a minimum?
min
k. What is the rate of change of this function over the interval [1,3]? -2

31. Check all the boxes that apply to the given functions as transformations to the parent function.

	Horizontal Shift Left	Horizontal Shift Right	Vertical Shift Up	Vertical Shift Down	Vertical Stretch	Vertical Shrink	Reflect
$f(x) = -4 \cdot 2^x$					✓		✓
$g(x) = 2(x-5)^2 - 1$		✓		✓	✓		
$h(x) = -(1/2)x + 4$			✓			✓	✓

32. For each equation below, write a new equation that shift the function down 6 and reflects over the x axis.
a. $f(x) = 3^x$ $-3^x - 6$

b. $g(x) = x^2$ $-x^2 - 6$

Glossary

Average Rate of Change: The change in the value of a quantity by the elapsed time. For a function, this is the change in the y-value divided by the change in the x-value for two distinct points on the graph.

Coefficient: A number multiplied by a variable in an algebraic expression.

Constant Rate of Change: With respect to the variable x of a linear function y = f(x), the constant rate of change is the slope of its graph.

Constant Term: A quantity that does not change its value.

Coordinate Plane: The plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. Each point in the coordinate plane can be specified by an ordered pair of numbers.

Continuous: Describes a connected set of numbers, such as an interval.

Discrete: A set with elements that are disconnected.

Domain: The set of x-coordinates of the set of points on a graph; the set of x-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

End Behaviors: The appearance of a graph as it is followed farther and farther in either direction.

Exponential Function: A nonlinear function in which the independent value is an exponent in the function, as in $y = ab^x$.

Exponential Model: An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

Horizontal Shift: A rigid transformation of a graph in a horizontal direction, either left or right.

Interval Notation: A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

Linear Function: A function with a constant rate of change and a straight line graph.

Linear Model: A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

Parameter: The independent variable or variables in a system of equations with more than one dependent variable.

Quadratic Equation: An equation of degree 2, which has at most two solutions.

Quadratic Function: A function of degree 2 which has a graph that "turns around" once, resembling an umbrella–like curve that faces either right–side up or upside down. This graph is called a parabola.

Range: The set of all possible outputs of a function.

Reflection: A transformation that "flips" a figure over a mirror or reflection line.

Root: The x–values where the function has a value of zero.

Slope: The ratio of the vertical and horizontal changes between two points on a surface or a line.

Variable: A letter or symbol used to represent a number.

Vertex: The maximum or minimum value of a parabola, either in terms of y if the parabola is opening up or down, or in terms of x if the parabola is opening left or right.

Vertical Translation: A shift in which a plane figure moves vertically.

X-intercept: The point where a line meets or crosses the x-axis. (x,0)

Y-Intercept: The point where a line meets or crosses the y-axis. (0,y)