

**Algebra 1 Unit 4  
Characteristics of Functions**

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This packet belongs to:

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**ALGEBRA**

Below are the formulas you may find useful as you take the test. However, you may find that you do not need to use all of the formulas. You may refer to this formula sheet as often as needed.

### Linear Formulas

#### **Slope Formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

#### **Linear Equations**

Slope-intercept Form:  $y = mx + b$

Point-slope Form:  $y - y_1 = m(x - x_1)$

Standard Form:  $Ax + By = C$

#### **Arithmetic Sequence Formulas**

Recursive:  $a_n = a_{n-1} + d$

Explicit:  $a_n = a_1 + (n - 1)d$

### Exponential Formulas

#### **Exponential Equation**

$$y = ab^x$$

#### **Geometric Sequence Formulas**

Recursive:  $a_n = r(a_{n-1})$

Explicit:  $a_n = a_1 \cdot r^{n-1}$

#### **Compound Interest Formula**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

### Quadratic Formulas

#### **Quadratic Equations**

Standard Form:  $y = ax^2 + bx + c$

Vertex Form:  $y = a(x - h)^2 + k$

#### **Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Average Rate of Change

The change in the  $y$ -value divided by the change in the  $x$ -value for two distinct points on a graph.

### Statistics Formulas

#### **Mean**

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

#### **Interquartile Range**

$$IR = Q_3 - Q_1$$

The difference between the first quartile and third quartile of a set of data.

#### **Mean Absolute Deviation**

$$\frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$$

The sum of the distances between each data value and the mean, divided by the number of data values.

Write your Questions here!

Whole Group Lesson

## Introduction: The Parent Functions

### Learning Targets:

- I can identify the visual differences between linear, quadratic and exponential functions.

F.LE.3

No Video

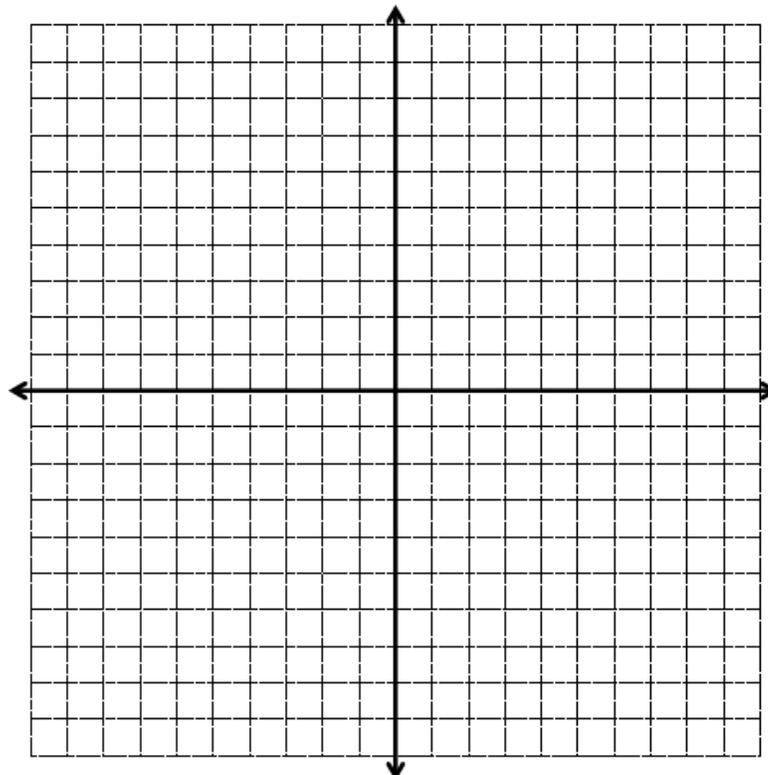
Complete the table

below.

Linear		Quadratic		Exponential	
$f(x) = x$		$g(x) = x^2$		$h(x) = 2^x$	
$x$	$f(x)$	$x$	$g(x)$	$x$	$h(x)$
-5		-5		-5	
-4		-4		-4	
-3		-3		-3	
-2		-2		-2	
-1		-1		-1	
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	

Draw and label each graph on the same coordinate plane.

Observations:



Linear

Quadratic

Exponential

# Transformations

## Vertical Shifts

Adding a value to the function (outside the parenthesis) shifts the graph \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

Subtracting a value from the function (outside the parenthesis) shifts the graph \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

## Horizontal Shifts

Adding a value to the x-value (inside the parenthesis) shifts the graph \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

Subtracting a value from the x-value (inside the parenthesis) shifts the graph \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

## Reflections

Multiplying the function by a negative \_\_\_\_\_ the graph across the \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

## Vertical Stretches and Shrinks

Multiplying the function by a value greater than 1 causes the graph to have a vertical \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

Multiplying the function by a value less than 1 causes the graph to have a vertical \_\_\_\_\_.

Ex:  $f(x) = 2^x$   
 $h(x) =$  \_\_\_\_\_

Write your Questions here!

Write your Questions here!

## 4-1: Transformations

### Learning Targets:

- I can determine the effect transformations have on a parent graph.

F.BF.3

Begin 4-1 Video 1

### Vocabulary:

- Exponential
- Horizontal Shift
- Linear
- Quadratic
- Reflection
- Vertical Shift

In this course we focus on three types of functions. Linear, Quadratic and Exponential. In the intro lesson, you saw what each of these three parent, or basic, functions look like. Now we are going to look at what happens when you change that function. Transformations of functions are very similar to what you do in Geometry when moving figures around the coordinate plane.

In this lesson, any reference to  $k$ , is a positive real number.

### Vertical Shifts (up & down)

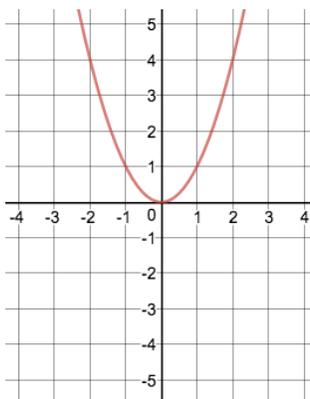
A shift may sometimes be referred to as a \_\_\_\_\_. If  $k$  is **added** to the \_\_\_\_\_ where we go from  $f(x)$  to  $f(x) + k$ , then the graph of  $f(x)$  will vertically shift \_\_\_\_\_ by  $k$  units. If  $k$  is **subtracted** from the \_\_\_\_\_ where we go from  $f(x)$  to  $f(x) - k$ , then the graph of  $f(x)$  will vertically shift \_\_\_\_\_ by  $c$  units.



In general, a vertical translation means that every ordered pair will go from  $(x, y)$  on the original  $f(x)$  to  $(x, y + k)$  on the newly transformed  $f(x) + c$  or to  $(x, y - k)$  on the newly transformed  $f(x) - k$ .

### Example 1:

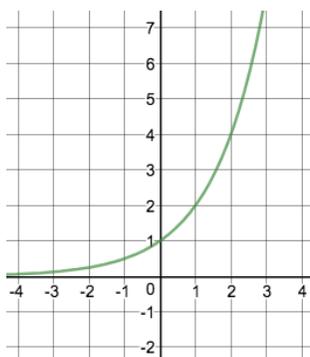
**1a)** Write an equation that would transform the parent function  $f(x) = x^2$  up 3 units.



**1b)** Write an equation that would transform the parent function  $f(x) = x^2$  down 4 units.

### Example 2

**2a)** What would happen to the graph of  $g(x) = 2^x$  if we transformed it to be  $h(x) = 2^x - 1.5$



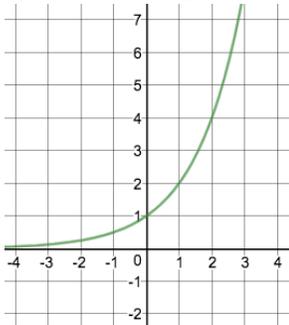
**2b)** What would happen to the graph of  $g(x) = 2^x$  if we transformed it to be  $h(x) = 2^x + 3$ ?

KEEP CALM  
AND  
DOWN SHIFT

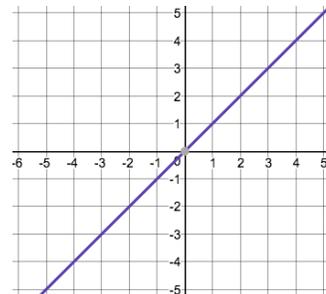
Write your Questions here!

**You try:**

1. Write an equation that would transform the parent function  $f(x) = 2^x$  up 4 units.



2. What would happen to the graph of  $y = x$  if we transformed it to be  $y = x - 2$ ?



Begin 4-1 Video 2

**Horizontal Shifts (left & right)**

If  $c$  is **added** to the \_\_\_\_\_ of the function where we go from  $f(x)$  to  $f(x + c)$ , then the graph of  $f(x)$  will be horizontally shifted to the \_\_\_\_\_  $c$  units. If  $c$  is **subtracted** from the \_\_\_\_\_ of the function where we go from  $f(x)$  to  $f(x - c)$ , then the graph of  $f(x)$  will be horizontally shifted to the \_\_\_\_\_  $c$  units.

Notice that anytime we are making a change “inside the parenthesis”, things shift in the opposite direction of what you would expect.

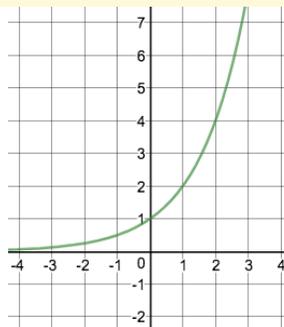
- Add to go \_\_\_\_\_.
- Subtract to go \_\_\_\_\_.

**Adding C** moves the function to the **left** (the negative direction).

**Why?** Well imagine you will inherit a fortune when your **age=25**. If you change that to **(age+4) = 25** then you will get it when you are 21. Adding 4 made it happen earlier.

**Example 3:**

**3a)** Write an equation that would transform the parent function  $y = 3^x$  to the left 2 points.



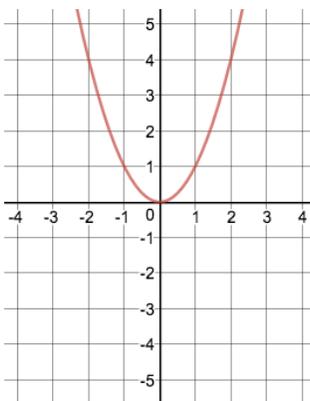
**3b)** Write an equation that would transform the parent function  $y = 3^x$  to the right 3 points.



Write your Questions here!

**Example 4:**

**4a)** What would happen to the graph of  $g(x) = x^2$  if we transformed it to be  $h(x) = (x + \frac{1}{3})^2$ ?



**4b)** What would happen to the graph of  $g(x) = x^2$  if we transformed it to be  $h(x) = (x - 3)^2$ ?

**You try:**

1. Write an equation that would transform the parent function  $f(x) = x^2$  right 1 unit.

2. What would happen to the graph of  $g(x) = 2^x$  if we transformed it to be  $h(x) = 2^{x+4}$ ?

Begin 4-1 Video 3

**Vertical stretching and shrinking**

If  $c$  is \_\_\_\_\_ to the function then the graph of the function will undergo a vertical \_\_\_\_\_ or \_\_\_\_\_.

- When the function becomes  $y = c * f(x)$  and  $0 < c < 1$ , a vertical \_\_\_\_\_ happens. This fractional  $c$  pulls the  $y$  values down towards the  $x$ -axis.
- When  $c > 1$  in the function  $y = c * f(x)$ , a vertical \_\_\_\_\_ occurs. The  $c$  multiplies the  $y$ -values and pushes them up and away from the  $x$ -axis.

In general, a vertical stretching or shrinking means that every point  $(x, y)$  on the graph is transformed to  $(x, cy)$  on the new graph.



When you vertically stretch a graph it is going to look tall and skinny.

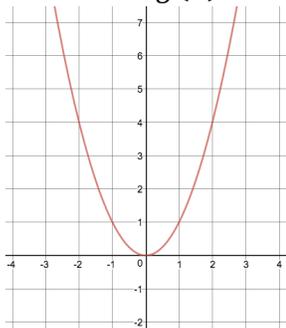
When you vertically shrink a graph, it will look short and wide.

To make sense of this, think of stretching or compressing a rubber band!

Write your Questions here!

**Example 5:**

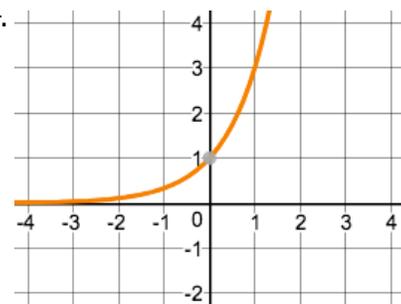
**5a)** Describe the transformation that would happen to the graph of  $f(x) = x^2$  if it was multiplied by a factor of 4 such that  $g(x) = 4x^2$ .



**You try:**

1. Write an equation that would vertically shrink the parent function  $f(x) = x^2$  by a factor of  $\frac{1}{2}$ .

**5b)** Write an equation that would vertically shrink the parent function  $f(x) = 3^x$  by a factor of  $\frac{1}{2}$ .



2. Describe the transformation that would happen to the graph of  $f(x) = 3^x$  if it was multiplied by a factor of 2 such that  $g(x) = 2 * 3^x$ .

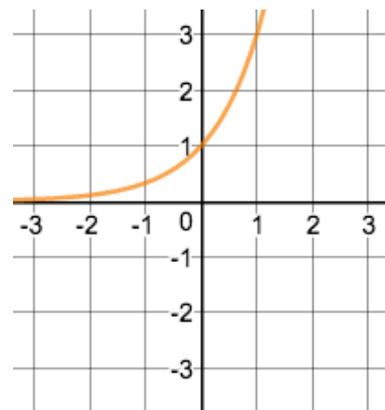


Begin 4-1 Video 4

**Reflection**

If the function is multiplied by -1, the graph of the function will undergo a reflection over the x-axis.

**Example 6:** Using the graph provided, show that multiplying the function  $f(x) = 3^x$  by -1 will reflect the graph across the x-axis.



**Example 7:** Describe how the graph of  $f(x) = 4^x$  has been transformed to form the graph of  $h(x) = -3(4)^{(x-2)} + 6$

- 
- 
- 
- 

**You try:**

1. Describe the transformation(s) on each function.

a.  $f(x) = \frac{1}{5}(2)^x - 4$

b.  $f(x) = 5(2)^{x-3}$

c.  $y = -2(x + 4)^2 + 4$

## Practice 4-1: Transformations

Describe the transformation on each function.

1.  $f(x) = 5x$

2.  $f(x) = -x + 3$

3.  $f(x) = -\frac{1}{2}x + 4$

4.  $f(x) = 2x - 5$

5.  $f(x) = -\frac{1}{5}x - 2$

6.  $f(x) = 5x + 3$

7.  $f(x) = \frac{1}{2}x + 4$

8.  $f(x) = -\frac{2}{5}x - 4$

9.  $f(x) = 2(x - 1)^2 - 2$

10.  $f(x) = 2x^2 + 3$

11.  $f(x) = \frac{1}{2}(x + 2)^2$

12.  $f(x) = -\frac{2}{3}(x)^2 - 4$

13.  $y = -(x - 3)^2 + 2$

14.  $y = -4(2)^{x+2} + 3$

15.  $y = \frac{3}{4}(x - 3)^2 - 4$

16.  $f(x) = 3x^2 - 3$

17.  $f(x) = \frac{1}{3}(2)^x + 2$

18.  $f(x) = 2(2)^x - 1$

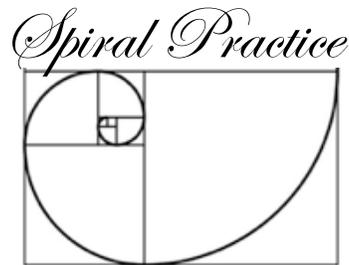
19.  $f(x) = 2^{x-2}$

20.  $f(x) = -2^x + 3$

**Write the equation of a function that has undergone the following transformations.**

21. A linear function that has been shifted to the right two.
22. A quadratic function that has been stretched by a factor of two and shifted down by four.
23. An exponential that has been reflected over the x-axis and shifted up 2.
24. A quadratic function that has been shrunk by a factor of  $\frac{1}{2}$ , shifted right 4, and up 3.
25. An exponential function that has been stretched by a factor of 2, reflected over the x-axis, shifted down two, and left one.

26. What is the area of a square with the length of a side equaling  $3a^5$ ?



27. What is the area of the rectangle with the width of  $6x^2$  and the length of  $12x^3$ ?

28. How many one-third cup servings are in 6 cups of pecans?

29. The perimeter of an isosceles triangle is 16 inches. The length of its base is 2 times the length of one of its other sides. In the equations below, x represents the length of each of its equal sides and y represents the length of its base. What are the three side lengths of the triangle?

$$2x + y = 16 \text{ and } y = 2x$$

Write your Questions here!

## 4-2: Rate of Change

**Learning Target:** I can find and interpret the rate of change of linear, exponential and quadratic functions.

F.IF.5

Begin 4-2 Video 1

### Vocabulary:

- Average Rate of Change
- Coefficient
- Constant Rate of Change
- Slope

The \_\_\_\_\_, or the \_\_\_\_\_, is a ratio describing how one quantity changes as another quantity changes. In algebra, we are typically comparing how the output changes to how the input changes.



**"I estimate that we are on a slope of about -0.625. What do you think?"**

*Average rate of change*

$$m = \left( \frac{y_2 - y_1}{x_2 - x_1} \right)$$

Formula Sheet!

If the value of the slope is \_\_\_\_\_, then we know that the rate of change is increasing over time.

If the rate of change \_\_\_\_\_ over time, you should expect a negative value.

There are many different ways you may be asked about rate of change.

### Example 1 - From ordered pairs:

**1a)** Find the slope of the line that goes through the points (-4,2) and (-6,4).

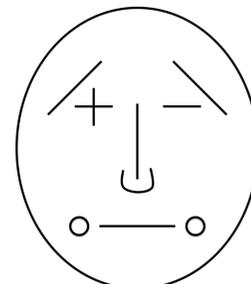
**1b)** Find the slope of the line that goes through the points (3,5) and (3,2).

### You try:

1. Find the slope of the line that goes through the points (3,-1) and (-2,5).

2. Find the slope of the line that goes through the points (3,2) and (1,2).

Remember



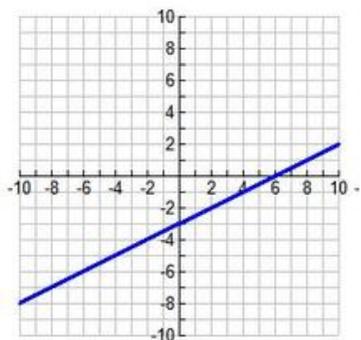
**MR. SLOPE GUY**

Write your Questions here!

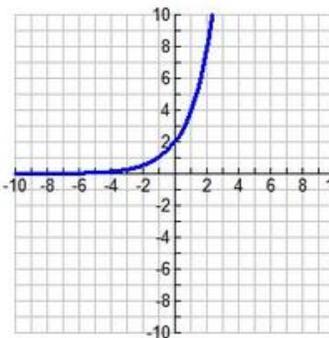
Begin 4-2 Video 2

Linear functions have a constant rate of change, meaning values increase or decrease at the same rate over a period of time. This also means that no matter where you look on the graph, the rate of change will be exactly the same.

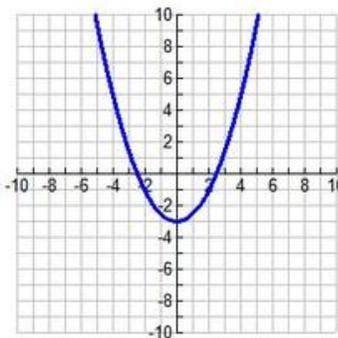
Both Exponential and Quadratic have variable rates of change. This means that the graph of the function will be steeper in some places than others. When calculating average rate of change for these functions, you will need to specify an interval.



Linear



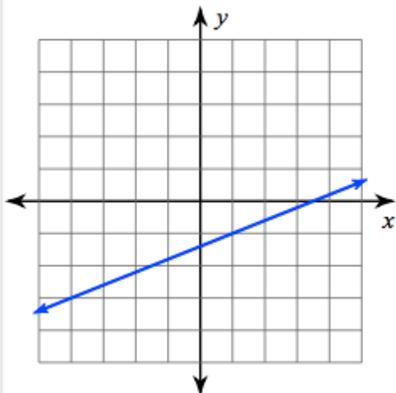
Exponential



Quadratic

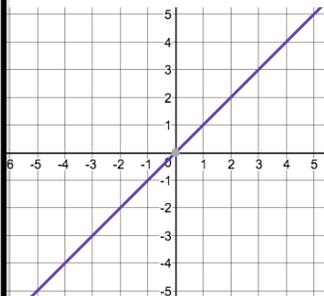
**Example 2 - From a graph:**

2a) Determine the slope of the line.

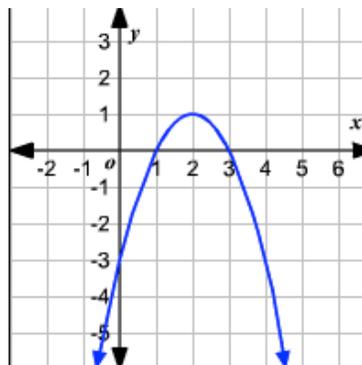


**You try:**

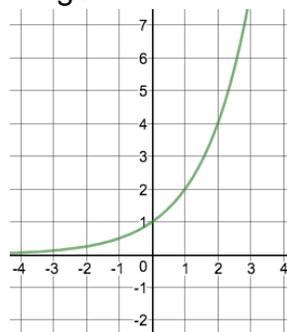
1. Determine the slope of the line.



2b) Determine the average rate of change over the interval  $[0, 2]$ .



2. Determine the average rate of change over the interval  $[1, 2]$ .



Write your Questions here!

### Begin 4-2 Video 3

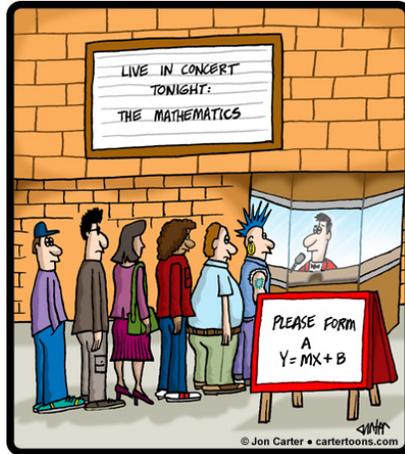
#### Example 3 – from an equation:

3a) Find the slope of the linear function,  $3x - 2y = 8$ .

3b) Determine the average rate of change from  $x = 1$  to  $x = 2$ , given the function,  $f(x) = x^2 + 2x - 7$ .

#### You try:

1. Find the slope of the line,  $y = -\frac{4}{3}x + 3$ .



2. Given the function,  $f(x) = 3^{x+1}$ , determine the average rate of change from  $x = -1$  to  $x = 1$ .

### Begin 4-2 Video 4

#### Example 4 – from a table:

Given the table of values for  $h(x)$ , what is the average rate of change from  $x = 6$  to  $x = 2$ ?

x	-2	0	2	4	6
$h(x)$	3	-1	3	15	35

#### Example 5 – from a scenario:

You are trying to decide which theater to go to for a Friday night movie. Habersham Hills charges \$7 for the movie ticket and \$5 per food item. Regal's prices are represented by the table.

Write an equation for Habersham Hills and Regal. Compare their rates of change and initial cost.

**Habersham Hills:**

**Regal:**

Number of food items with ticket (x)	Price in \$ (y)
0	8
1	12
2	16
3	20
4	24

Which theater is cheaper if you want to see the movie and also get a drink and popcorn?



Write your  
Questions here!

**You try:**

1. The average price for a ticket to a movie theater in North America for selected years is shown in the table below.

Find the rate of change from 2010 to 2015. Then find the rate of change from 1995 to 2005? How do the rates of change compare?

Year	1985	1990	1995	2000	2005	2010	2015
Price (\$)	3.90	4.20	4.50	4.80	5.10	5.40	5.70

2. Given the table of values for  $f(x)$ , what is the average rate of change from  $x = 1$  to  $x = 3$ ?

$x$	-1	0	1	2	3
$f(x)$	-0.25	-1	-4	-16	-64

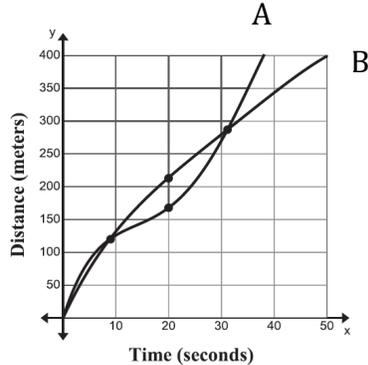
# Practice Makes Better

Write your Questions here!

## Practice 4-2: Rate of Change

Use the information provided to answer the questions below.

1. Below is the graph and table for 2 runners running the 400 meter hurdles race.



Time	Runner A	Runner B
0	0	0
9	120	120
20	168	213
31	287	287

a) Which runner has a faster average speed for the first 9 seconds?

b) Which runner has a faster average speed from 9 to 20 seconds?

e) Which runner wins the race?  
How do you know?

c) Which runner has a faster average speed from 20 to 31 seconds?

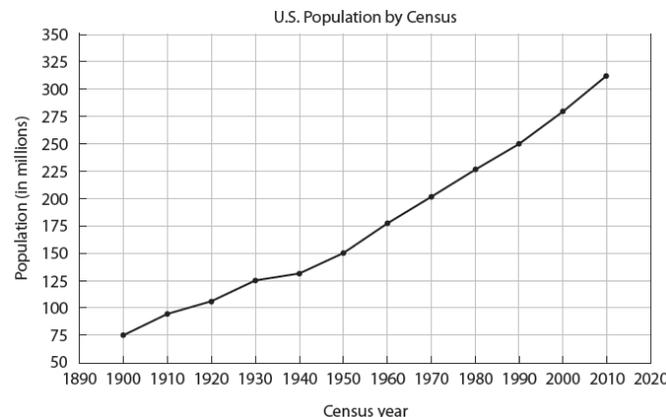
d) Which runner has a faster average speed from 9 to 31 seconds?

Hint:  
Speed is another name for average rate of change!

2. The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.

a) What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

b) Which 10-year time periods have the highest and the lowest rates of change? How did you find these?



Write your Questions here!

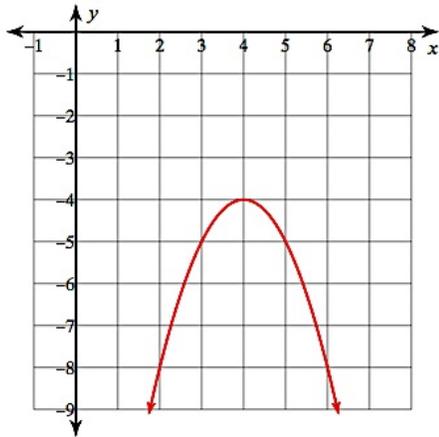
3. Find the rate of change of Pete's height from 3 to 5 years.

Time (years)	1	2	3	4	5	6
Height(in.)	27	35	37	42	45	49

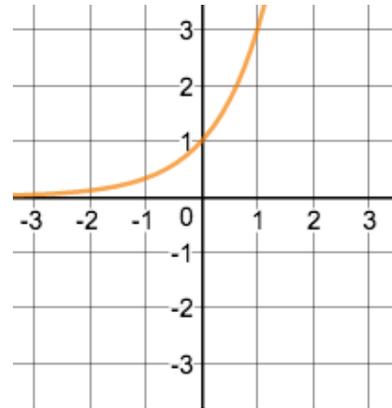
4. For  $f(x) = -6x - 2$ , find the rate of change on the interval  $[-2, 4]$ .

**Find the rate of change (slope) of each graph over the given interval.**

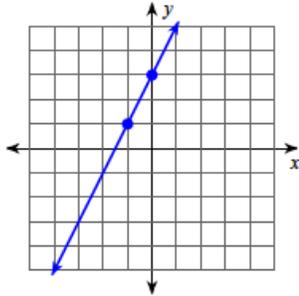
5. Find the rate of change over the interval  $[2, 4]$



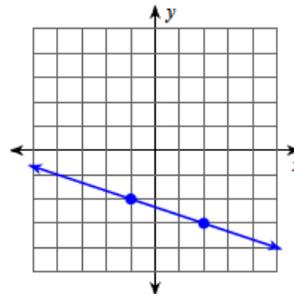
6. Find the average rate of change over the interval  $[0, 1]$



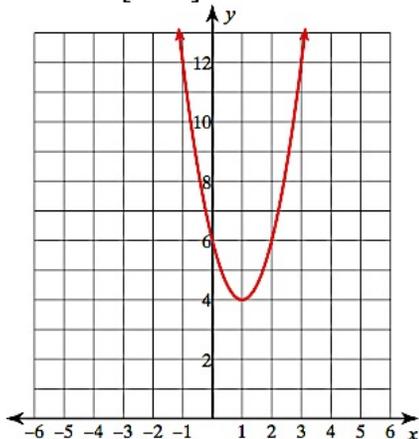
7. Find the slope of the line.



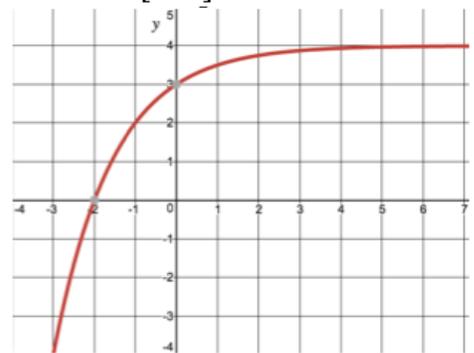
8. Find the slope of the line.



9. Find the rate of change over the interval  $[-1, 1]$



10. Find the average rate of change over the interval  $[-2, 0]$



Write your Questions here!

**Find the rate of change (slope) through each pair of points.**

11.  $(-14, -8), (-10, -7)$

12.  $(5, 4), (8, 5)$

13.  $(-18, -17), (-17, 16)$

14.  $(13, 7), (13, 10)$

**Find the rate of change (slope) of each line.**

15.  $y = -5x + 5$

16.  $y = 5x + 5$

17.  $3x + y = 0$

18.  $x = 4$

**Find the rate of change (slope) of each function over the given interval.**

19. Determine the average rate of change from  $x = -2$  to  $x = 1$ , given the function,  $f(x) = x^2 + 4x - 3$ .

20. Determine the average rate of change from  $x = 0$  to  $x = 2$ , given the function,  $f(x) = 2(3)^x - 1$ .

21. Determine the average rate of change from  $x = 0$  to  $x = 2$ , given the function,  $f(x) = 5^x - 2$ .

22. Determine the average rate of change from  $x = -1$  to  $x = 1$ , given the function,  $f(x) = x^2 - 6x + 9$ .

23. Given the table of values for  $f(x)$ , what is the average rate of change from  $x = -2$  to  $x = 1$ ?

x	-2	-1	0	1	2
f(x)	9	4	1	0	1

24. Given the table of values for  $g(x)$ , what is the average rate of change from  $x = 0$  to  $x = 4$ ?

x	-2	0	2	4	6
g(x)	21	7	9	27	61

Write your Questions here!

25. Given the table of values for  $h(x)$ , what is the average rate of change from  $x = 5$  to  $x = 9$ ?

$x$	1	3	5	7	9
$h(x)$	-4	6	24	50	84

26. Given the table of values for  $j(x)$ , what is the average rate of change from  $x = -4$  to  $x = -2$ ?

$x$	-5	-4	-3	-2	-1
$j(x)$	3	0	-1	0	3

27. Find the rate of change of the cost per minute.

Minutes ( $x$ )	Cost in Dollars ( $f(x)$ )
0	110.00
30	120.50
60	131.00
90	141.50
120	152.00

28. Find the rate of change of revolutions per second.

Seconds ( $x$ )	Revolutions ( $f(x)$ )
0	0
2	6
4	12
10	30
15	45

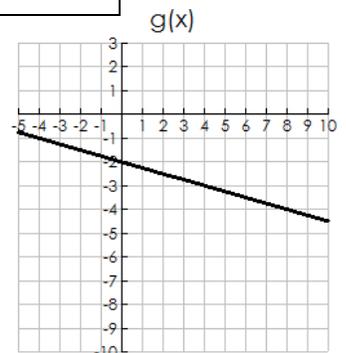
29. For the following two functions, write the equations of each and complete the chart using  $<$ ,  $>$ , or  $=$  to compare them.

Characteristic of $f(x)$	$<$ , $>$ , or $=$	Characteristic of $g(x)$
y-intercept of $f(x) =$		y-intercept of $g(x) =$
$f(4) =$		$g(4) =$
Rate of change of $f(x) =$		Rate of change of $g(x) =$

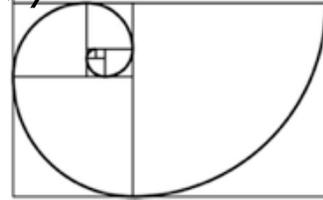
$x$	$f(x)$
-3	11
-1	7
1	3
3	-1
5	-5

$f(x) =$

$g(x) =$



## *Spiral Practice*



30. A pet store had 4 cats to feed. If they only had one-fifth of a bag of cat food and each cat got the same amount, what fraction of the bag would each cat get?

31. Solve for  $x$  in the following equation  $5(x + 2) = \frac{3}{5}(5 + 10x)$

32. Kaya and Tad started with the same number of baseball cards in their collections. Kaya collected 3 cards per week and now has 29 cards. Tad collected 2 cards per week and now has 20 cards. Let  $x$  represent the number of cards they began with, and let  $y$  represent the number of weeks. Write a linear system that represents this situation. How many weeks did they collect cards?

**Review your practice and notes to prepare for the mastery check.**

Write your Questions here!

### 4-3: Domain and Range

#### Learning Target:

- I can identify the domain and range of a function.

F.IF.1

Begin 4-3 Video 1

#### Vocabulary:

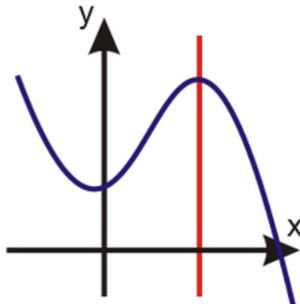
- Continuous
- Discrete
- Domain
- Interval Notation
- Range

The relationship between two quantities can be shown by a set of ordered pairs called a \_\_\_\_\_.

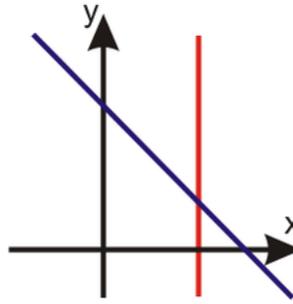
In the very first lesson of the course, we talked about the fact that a function was a special relation in which each input ( $x$ ) has exactly \_\_\_\_\_ output ( $y$ ). The majority of the relations we will deal with throughout the remainder of the year will be functions. But how do you know if you're dealing with a function or just a relation?

#### Two ways to determine a function:

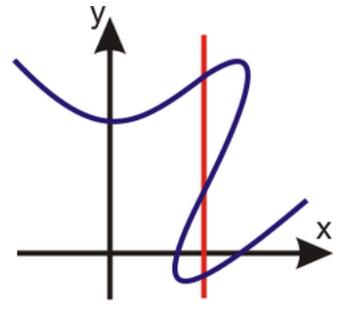
- Scan the \_\_\_\_\_
  - If an  $x$  value repeats, you \_\_\_\_\_ a function!
- Vertical Line Test
  - If any \_\_\_\_\_ line passes through more than one point of the graph, then the relation is **not** a function.



This is a function



This is a function



This is NOT a function

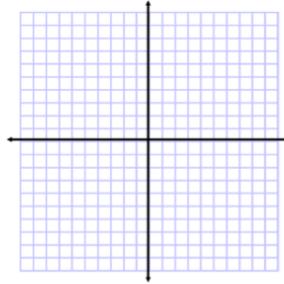
Relations can be expressed in several different ways.

The relation  $\{(2, 3), (4, 7), (6, 8)\}$  can be displayed in the following ways:

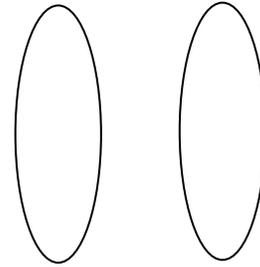
An x/y table

X	Y

A graph



Using Mapping



The \_\_\_\_\_ of a relation is the set of inputs (or x-values) of the ordered pairs.

- “What x-values were used in this graph? How far left does it go? How far right?”

The \_\_\_\_\_ of a relation is the set outputs (or y-values) of the ordered pairs.

- “What y-values were used in this graph? How far down does it go? How far up?”

**Example 1:**

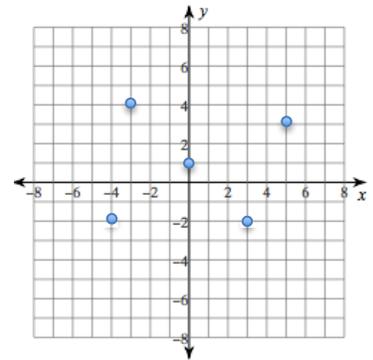
$\{(-1, 4), (2, 6), (0, 4), (-7, 1)\}$

Domain:

Range:

Function?

**Example 2:**



Domain:

Range:

Function?

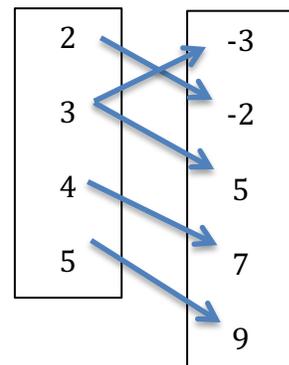
**Helpful Hint**

**There are several different ways to describe the variables of a function.**

Independent Variable	Dependent Variable
x-values	y-values
Domain	Range
Input	Output
x	$f(x)$

**Example 3:**

Find the domain and range of the following relation.



Domain:

Range:

Function?

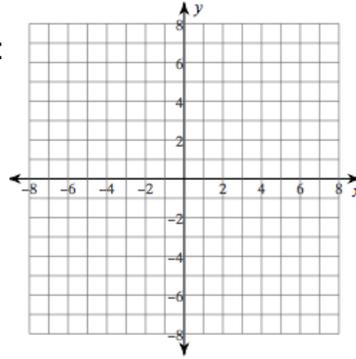
Write your Questions here!

Write your Questions here!

**Example 4:**

Students who take the AP Government exam can score a 1, 2, 3, 4, or 5. Last year, two students scored a 1, no students scored a 2, three students scored a 3, seven students scored a 4 and seven students scored a 5. Write these scores as a set of ordered pairs and plot them on the coordinate plane.

Ordered Pairs:



Domain:

Range:

Function?

Should we connect these points with a line?

**You try:**

Identify the domain and range of each relation.

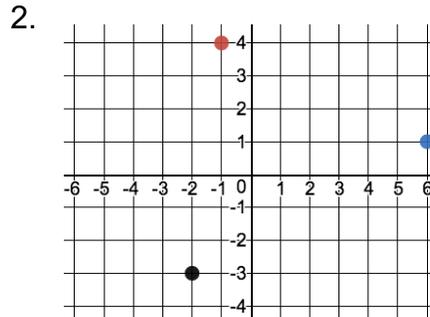
1.

x	3	-1	-6	1	4
y	-4	7	-8	11	13

Domain:

Range:

Function?



Domain:

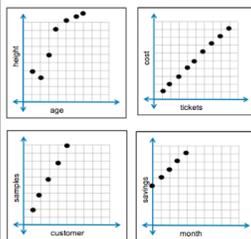
Range:

Function?

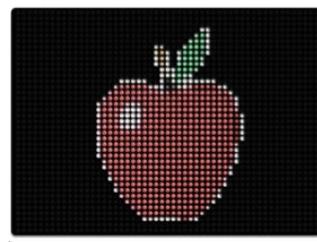
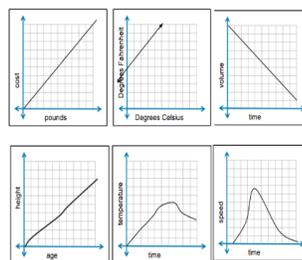
Begin 4-3 Video 2

All of the previous examples have had distinct points. We call relations like this discrete. But what if a relation has an infinite number of points? This is called a continuous relation.

**DISCRETE**



**CONTINUOUS**



LITE BRITE  
discrete



ETCH-A-SKETCH  
continuous

Write your Questions here!

There are a couple of different notations you can use to describe domain and range of continuous relations.

### Set Notation

- Uses inequality symbols like  $<$ ,  $>$ ,  $\leq$ , and  $\geq$ .
- When a boundary is included in the domain or range, we use either the \_\_\_\_\_ symbol.
  - Why? Because our domain or range can equal that value!
- If the boundary is not included in the domain or range, we use either the \_\_\_\_\_ symbol.
  - Why? Because the domain or range cannot equal that value.
- When either \_\_\_\_\_ is our bounds, we use \_\_\_\_\_.
  - Why? You can never actually reach infinity!

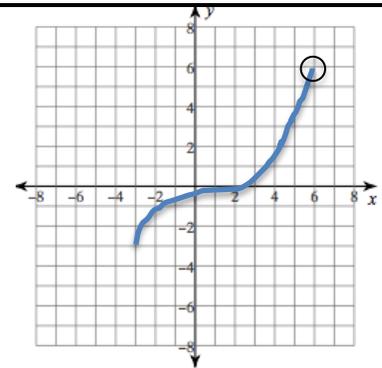
### Interval Notation

- Uses brackets and parentheses.
- We use brackets when the boundary is \_\_\_\_\_.
- We use parentheses when the boundary is \_\_\_\_\_.
- Use parentheses when a boundary is \_\_\_\_\_ or \_\_\_\_\_.
- Why? You can never actually reach infinity!

#### Example 5:

Give the Domain and range of this continuous relation in set and interval notation.

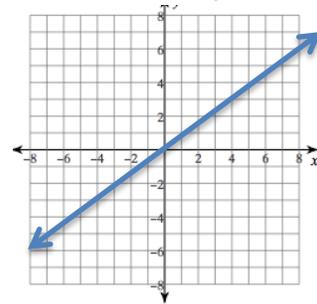
	Set Notation	Interval Notation
Domain		
Range		



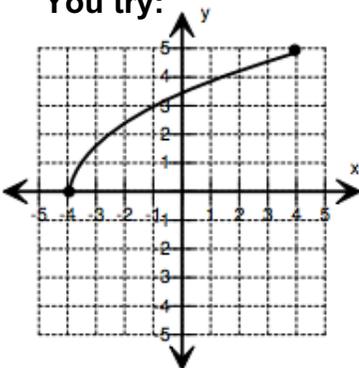
#### Example 6:

Give the Domain and range of this continuous relation in set and interval notation.

	Set Notation	Interval Notation
Domain		
Range		

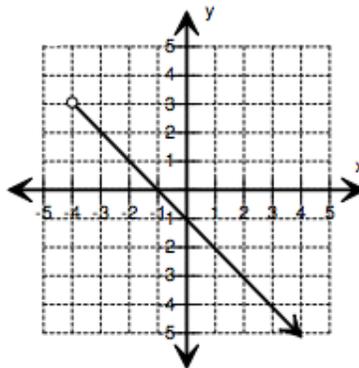


#### You try:



Domain : \_\_\_\_\_

Range : \_\_\_\_\_



Domain : \_\_\_\_\_

Range : \_\_\_\_\_

How many ways can we say the same thing?

1.  $-\infty < x < \infty$
2. All real numbers
3.  $(-\infty, \infty)$
4.  $\mathbb{R}$

Write your Questions here!

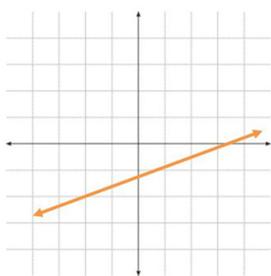
## Begin 4-3 Video 3

GOOD NEWS, EVERYONE!



*Math is all about patterns...* and the three functions we focus on in Algebra 1, (Linear, Quadratic and Exponential) have some great patterns that make domain and range so much easier to deal with!

### Linear:



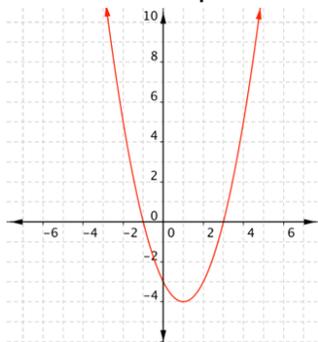
An unbound (meaning no restrictions) linear function will \_\_\_\_\_ have a domain of  $\mathbb{R}$  and a range of  $\mathbb{R}$ .

- A domain of all real numbers means that you can plug in any number you can think of and the function will give you an answer.
- A range of all real numbers means that eventually, your answers (outputs) will hit every number you can think of.

**Example 7:** Domain: \_\_\_\_\_ Range: \_\_\_\_\_

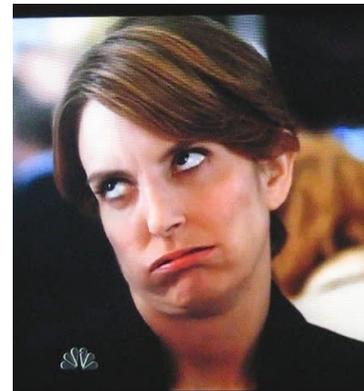
### Quadratic:

An unbound quadratic function will have a domain of  $\mathbb{R}$ , the range will be restricted based upon the \_\_\_\_\_.



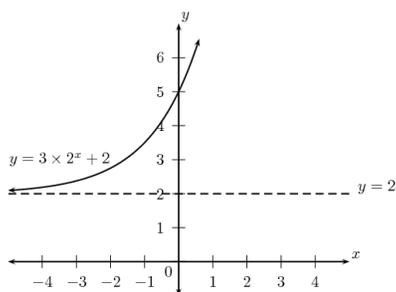
- Domain of all real numbers = plug in anything you want!
- The range (or possible outputs) of a quadratic function will be limited by the minimum or maximum value (vertex). This is the point at which the parabola \_\_\_\_\_. We will study these in more detail in a later lesson.

**Example 8:** Domain: \_\_\_\_\_ Range: \_\_\_\_\_



### Exponential:

An unbound exponential function will have a domain of  $\mathbb{R}$ . The range will depend on the \_\_\_\_\_.



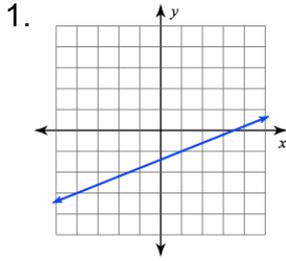
- Domain of all real numbers = Every x is a possibility!
- The range in exponentials is restricted because these functions have line that they cannot cross. This line is called an asymptote and changes from one function to another. We will study these in more detail in a later lesson.

**Example 9:** Domain: \_\_\_\_\_ Range: \_\_\_\_\_

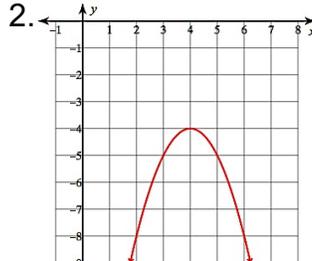
**WARNING**  
The following hints have their limitations. Always read instructions carefully!

Write your Questions here!

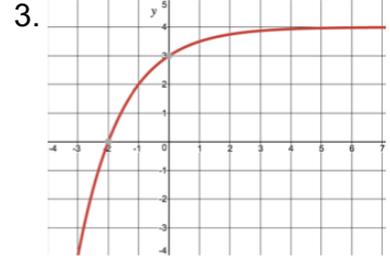
**You try:** Find the domain and range of each of the following functions.



D: \_\_\_\_\_ R: \_\_\_\_\_



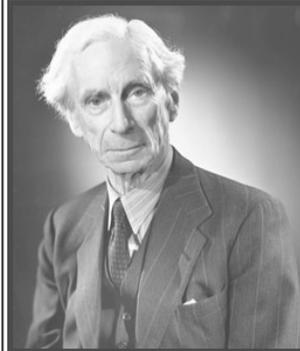
D: \_\_\_\_\_ R: \_\_\_\_\_



D: \_\_\_\_\_ R: \_\_\_\_\_

Begin 4-3 Video 4

Those patterns are awesome. But real life has limitations. Be prepared to **not** write “all real numbers” when dealing with a word problem.



Real life is, to most men, a long second-best, a perpetual compromise between the ideal and the possible; but the world of pure reason knows no compromise, no practical limitations, no barrier to the creative activity.

(Bertrand Russell)

*Historical Note:*

Bertrand Russell was a British philosopher and mathematician that was born in 1872 and died in 1970.

**Example 10**

Zach is making friendship bracelets to fund his spring break plans. He is renting a booth at the Mt. Laurel Festival to sell them and is charging \$6 per bracelet. Zach’s profit,  $y$ , in dollars, for selling  $x$  bracelets can be modeled by the function  $y = 6x - 225$ . What is the domain of this function?

**Example 11**

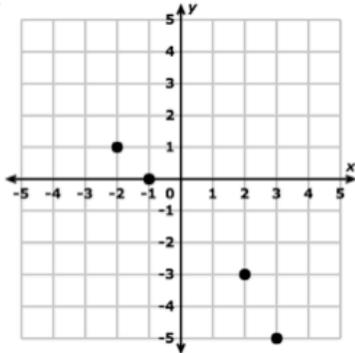
A tub drains at a rate of 3 gallons every 20 seconds. The typical tub holds approximately 30 gallons of water. Let  $y$  be the amount of water in the tub for a given  $x$  amount of time in seconds. Describe the domain and range of the function.

## Practice 4 – 3: Domain & Range

Find the domain and range for each relation and determine whether it is a function.

Write your Questions here!

1.

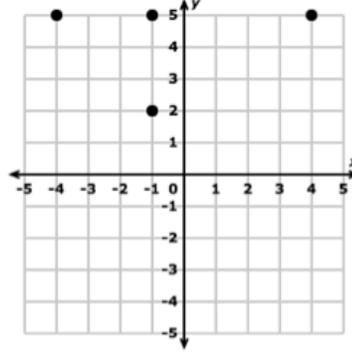


Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

2.

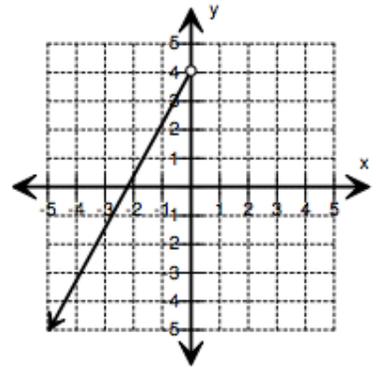


Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

3.

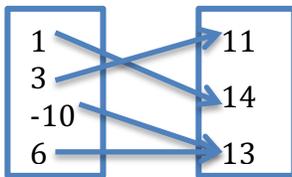


Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

4.

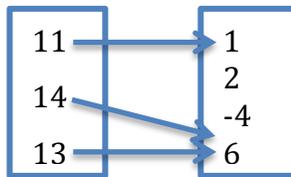


Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

5.

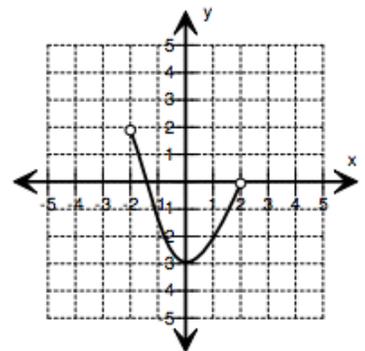


Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

6.



Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

7.  $\{(3,2), (5,7), (1,4), (9,2), (3,7)\}$

Domian: \_\_\_\_\_

Range: \_\_\_\_\_

Function? \_\_\_\_\_

8.  $\{(6,2), (3,5), (9,0), (5,7), (8,1)\}$

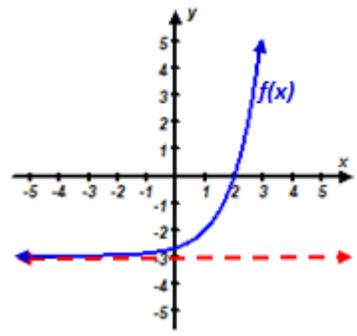
Domian: \_\_\_\_\_

Range: \_\_\_\_\_

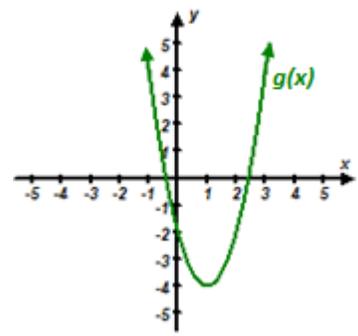
Function? \_\_\_\_\_

Determine the Domain and Range of each of the following graphed functions (using Interval and Set Notations).

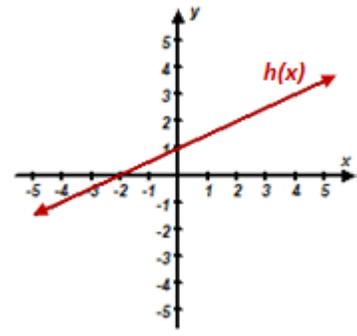
Write your Questions here!



9.  
11.



10.



Domain (INTERVAL): \_\_\_\_\_

Domain (INTERVAL): \_\_\_\_\_

Domain (INTERVAL): \_\_\_\_\_

Domain (SET): \_\_\_\_\_

Domain (SET): \_\_\_\_\_

Domain (SET): \_\_\_\_\_

Range (INTERVAL): \_\_\_\_\_

Range (INTERVAL): \_\_\_\_\_

Range (INTERVAL): \_\_\_\_\_

Range (SET): \_\_\_\_\_

Range (SET): \_\_\_\_\_

Range (SET): \_\_\_\_\_

12. What values of  $x$  would make this relation not be a function?  
(3,2); (x,4); (5,7); (8,-1)

13. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of  $[-\infty, \infty]$** ?

14. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of  $(2, \infty)$** ?

15. If we only considered the functions LINEAR, QUADRATIC, and EXPONENTIAL, which is the only one that could have a **range of  $[-5, \infty)$** ?

16. An author is selling autographed copies of his book at a stand in a bookstore in the mall and charging \$12 per copy. The author brought a total of 40 books with him to sell at his stand. If the function  $p(x) = 12x$  represents the gross profit the author could make during the time he is sitting at the stand, determine the appropriate domain and range.



**Domain:** \_\_\_\_\_

**Range:** \_\_\_\_\_

17. A postal company delivers packages based on their weight but will not ship anything over 50 pounds. The company charges \$0.50 per pound to deliver the package anywhere in the United States. If we consider this situation a function where the number of pounds,  $x$ , is the independent variable and the cost in dollars,  $y$ , is the dependent variable determine the domain and range.

Write your Questions here!

**Domain:**

**Range:**



18. A limousine company rents their limousine by the hour. The company charges \$85 per hour. The minimum time is 2 hours and a maximum of 12 hours. If we consider this situation a function where the number of hours,  $x$ , is the independent variable and the cost in dollars of renting the limousine,  $y$ , is the dependent variable determine the domain and range.

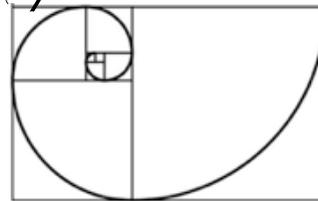
**Domain:**

**Range:**



19. Annita makes cupcakes to sell at a bake sale. She fills each box with 4 cupcakes. How many boxes will Annita need to hold 104 cupcakes?

*Spiral Practice*



**This WILL be on your mastery check!**

20.  $f(x) = \frac{4}{7}x + 8$  find the value of  $x$  if  $f(x) = 28$

21.  $f(x) = 6(4x^2 - 3x + 2) - (3x - 6)$  find  $f(0)$

22. Put in slope-intercept form and tell the slope and y-intercept  
 $2x - 5y = 10$

**Review your practice and notes to prepare for the mastery check.**

Write your Questions here!

### 4-4: Intercepts

#### Learning Targets:

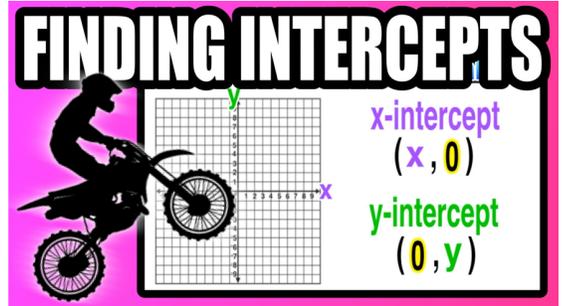
- I can determine the x and y intercepts of functions.

F.IF.4



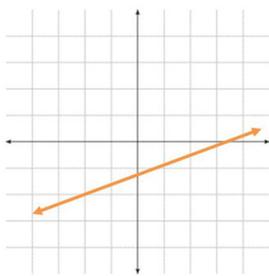
**Vocabulary:**  
 - Root  
 - x-intercept  
 - y-intercept

The graphical concept of x- and y-intercepts is pretty simple. The x-intercepts are where the graph crosses the           , and the y-intercepts are where the graph crosses the           .



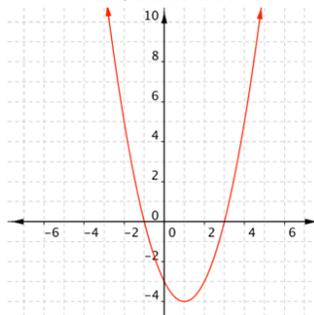
#### Example 1:

Linear



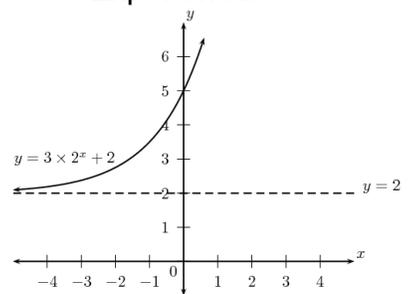
x-intercept(s):  
 y-intercept:

Quadratic



x-intercept(s):  
 y-intercept:

Exponential



x-intercept(s):  
 y-intercept:

Once again, mathematics offers some patterns that we should take note of.

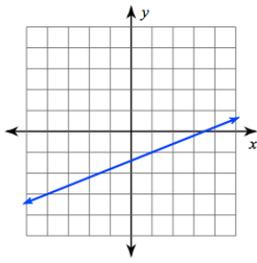
	How many can I expect to see?		
	Linear	Quadratic	Exponential
x-intercept	One	Zero, One, Two	Zero, One
y-intercept	One	One	One



**You try:** Identify the x- and y-intercepts in the functions below.

1.

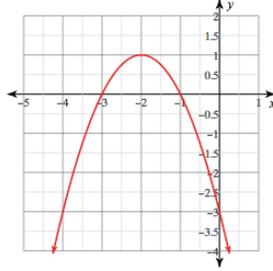
x-



int(s): \_\_\_\_\_

y-int: \_\_\_\_\_

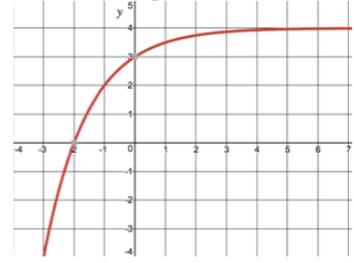
2.



x-int(s): \_\_\_\_\_

y-int: \_\_\_\_\_

3.



x-int(s): \_\_\_\_\_

y-int: \_\_\_\_\_



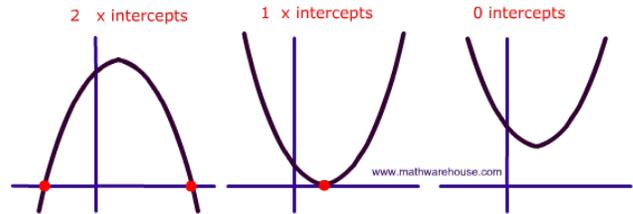
Finding intercepts from a graph is very straightforward. The problems start when we try to deal with intercepts algebraically.

**x-intercept:**

The x-coordinate of the point at which the graph of an equation crosses the x-axis, where \_\_\_\_\_.

Steps to find the x-intercept:

1. Plug in  $y = 0$ .
2. Always write as an ordered pair. \_\_\_\_\_



**Example 2:** Find the x-intercept of the given equation.

$y = 5x + 4$

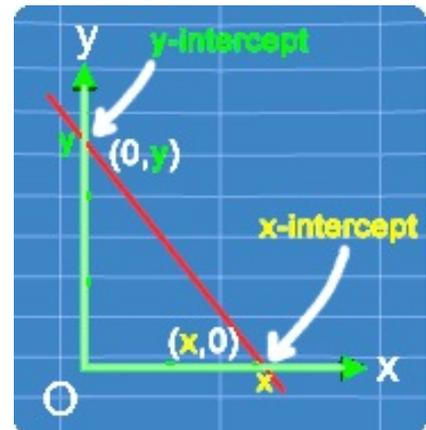
**Note:** At this point, you are only prepared for finding x-intercepts of Linear functions algebraically. Quadratic and Exponential x-intercepts can be found graphically now. Algebraic methods will be covered in later units.

**Example 3:** Find the x-intercept of the given equation.

$-5x + y = 4$

**You try:** Find the x-intercept of the given equations.

1.  $y = -2x + 4$
2.  $4x + 3y = 4$



Write your Questions here!

Begin 4-4 Video 3



In a Linear Equation, the y-intercept can be found by putting the function in slope-intercept form and finding the "b".

$$y = mx + b$$

**y-intercept:**

The y-coordinate of the point at which the graph of an equation crosses the y-axis, where \_\_\_\_\_.

Steps to find the y-intercept:

1. Plug in  $x = 0$ .
2. Always right as an ordered pair. \_\_\_\_\_

**Example 4:** Find the y-intercept of the given equation.

$$y = 5x + 4$$

**Example 5:** Find the y-intercept of the given equation.

$$3x^2 - 5x + 2 = y$$

**Example 6:** Find the y-intercept of the given equations.

$$y = -2(2)^x + 4$$

**You try:** Find the y-intercept of the given equations.

1.  $y = -2x + 4$

2.  $y = 4x^2 + 3x - 4$

3.  $3^x - 4 = y$

Begin 4-4 Video 4

Interpreting the intercepts.

Example 7: Zoo Atlanta recently celebrated the birth of two new baby pandas!

Mochi the panda cub has been measured and weighed each week since she was born.

Weeks	Weight
0	1
1	5
2	9
3	13

Mochi's brother is Kappa. His weight has been charted on the graph below.



What is the y-intercept for Mochi and for Kappa? What is the significance of these values for the baby pandas?

Neither panda has an x-intercept. Why? What would that mean in the context of this problem?

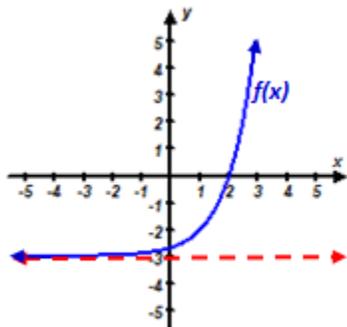
Write your Questions here!

# Practice Makes Better

## Practice 4 – 4: Intercepts

Find the x- and y-intercepts for each function.

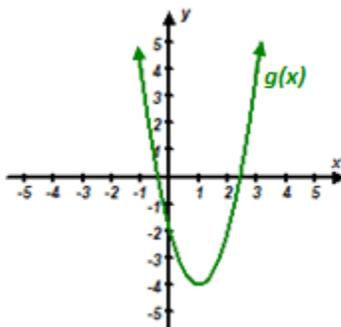
1.



x-int(s): \_\_\_\_\_

y-int: \_\_\_\_\_

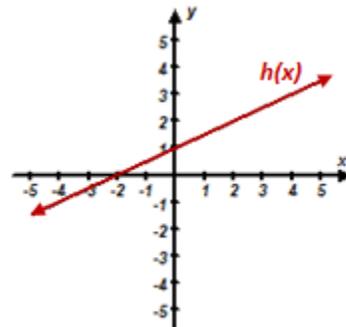
2.



x-int(s): \_\_\_\_\_

y-int: \_\_\_\_\_

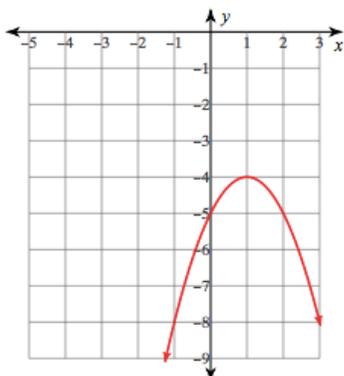
3.



x-int(s): \_\_\_\_\_

y-int: \_\_\_\_\_

4.



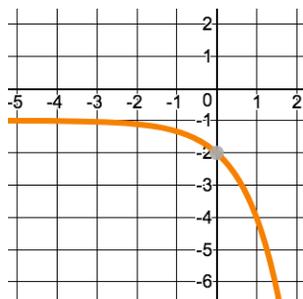
x-int(s):

\_\_\_\_\_

y-int:

\_\_\_\_\_

5.



x-int(s):

\_\_\_\_\_

y-int:

\_\_\_\_\_

Find the x-intercept of the following functions.

6.  $3x + 7y = 12$

7.  $y = \frac{1}{2}x - 7$

8.  $y = -5x + 3$

9.  $y = 0.2x + 6$

10.  $y = 4x + 3$

11.  $2x - y = -10$

Find the y-intercept of the following functions.

12.  $3x + 7y = 12$

13.  $y = 4^{x+3}$

14.  $y = -2(3^x) + 7$

15.  $y = -2(x - 4)^2 + 6$

16.  $y = 4x + 3$

17.  $y = 12x^2 - 5x + 3$

Write your Questions here!

18. Two contestants on The Biggest Loser are Valerie and Oscar. Their weight loss progress is shown below.

Valerie's weight loss is shown by this function, where  $W$  is her weight in pounds and  $t$  is the time in weeks.

$$W = 235 - 2.5t$$

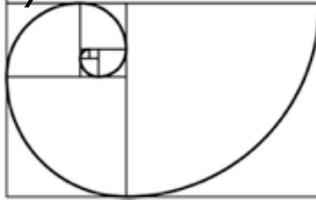
Oscar's weight loss is tracked in the table below.

Weeks	0	2	5	6
Weight	247	243	237	235

What is the y-intercept for Valerie and for Oscar? What is the significance of these values for the contestants? Who is losing weight faster?

What mathematical term would we use to describe the amount of weight they are losing each week?

*Spiral Practice*



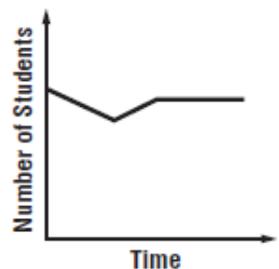
Add or subtract the Polynomials

19.  $(4a^2 - 2a + 1) - (a^3 - 2a + 3)$

20.  $(3x^3 + 4x + 14) + (-4x^2 + 21)$

21.  $(x^2 + 11xy - 3y^2) + (-2x^2 - xy + 4y^2)$

22. The graph below shows the number of students enrolled at Edison Junior High School. Describe the change in the number of students over time.



**Review your practice and notes to prepare for the mastery check.**

Write your Questions here!

## 4-5: Intervals of Positive & Negative Intervals of Increasing & Decreasing

Learning Targets:

- I can identify the intervals of increase & decrease of a function.
- I can identify intervals of positive & negative of a function.

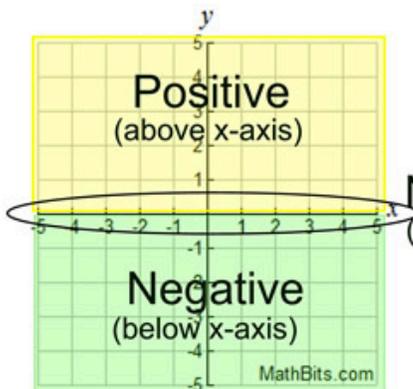
F.IF.4

Begin 4-5 Video 1

Positive/Negative

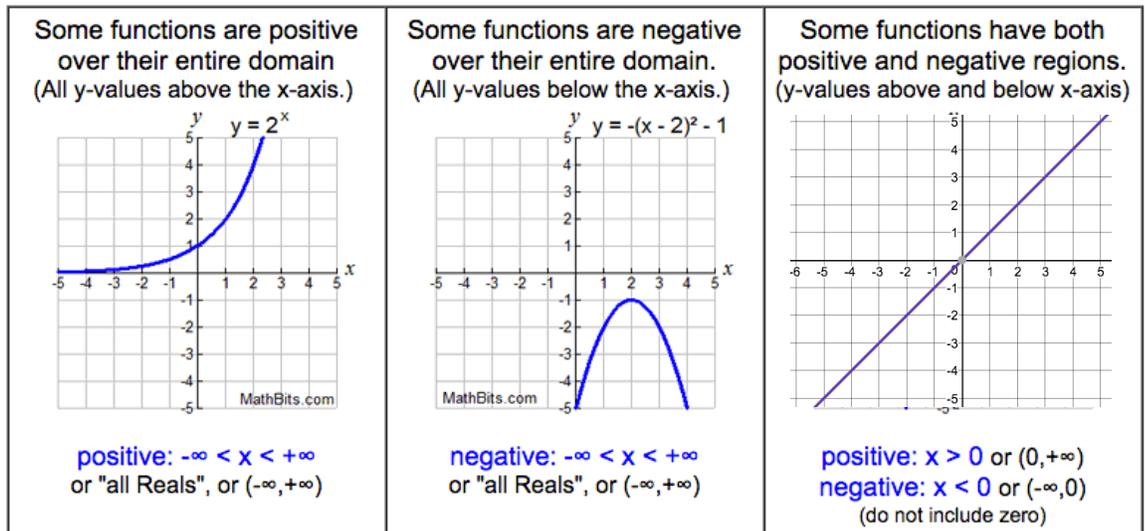
Not Zero!

y-values positive  
or  
y-values negative



- The positive regions of a function are those intervals where the function is \_\_\_\_\_ the x-axis. It is where the \_\_\_\_\_ are positive (not zero).
- The negative regions of a function are those intervals where the function is \_\_\_\_\_ the x-axis. It is where the y-values are \_\_\_\_\_ (not zero).
- y-values that are on the x-axis are neither positive nor negative. The x-axis is where \_\_\_\_\_.

Example 1:



Revisit 3-3  
Video 2 if  
you need a  
refresher!

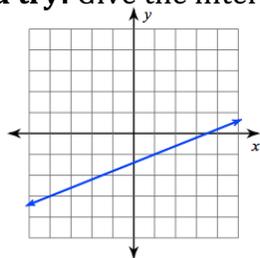
### Secret to Finding the Intervals!

The secret to correctly stating the intervals where a function is positive or negative is to remember that the **intervals ALWAYS** pertain to the locations of the **x-values**. Think of reading the graph from left to right along the **x-axis**. Do NOT read numbers off the y-axis for the intervals. **Stay on the x-axis!**

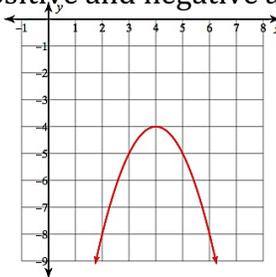
Write your Questions here!

**You try:** Give the intervals of positive and negative as applicable.

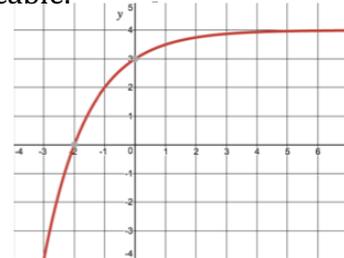
1.



2.



3.



Begin 4-5 Video 2

## Increasing/Decreasing

When looking for sections of a graph that are increasing or decreasing, be sure to look at (or "read") the graph **from left to right**.

**Increasing:** A function is *increasing*, if as  $x$  increases (reading from left to right),  $y$  also                     .

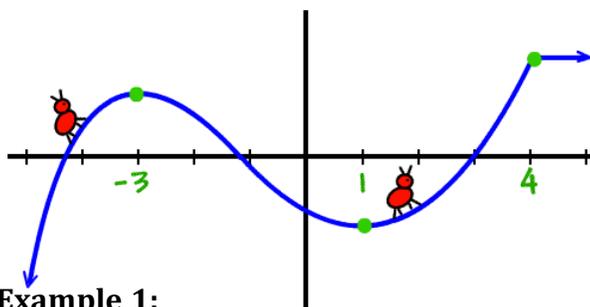
- In plain English, as you look at the graph, from left to right, the graph goes                     .
- The graph has a                      slope.

**Decreasing:** A function is *decreasing*, if as  $x$  increases (reading from left to right),  $y$  decreases.

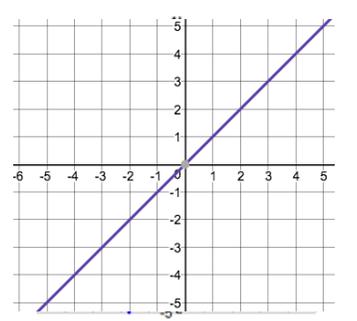
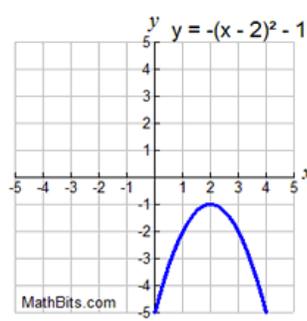
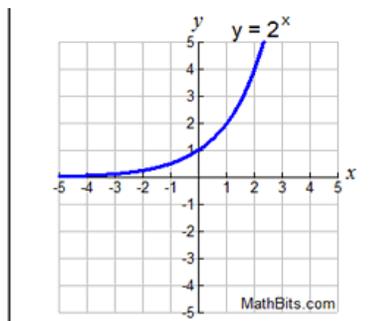
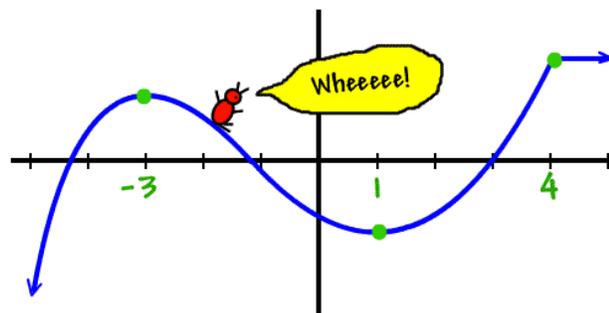
- In plain English, as you look at the graph, from left to right, the graph goes down-hill.
- The graph has a *negative slope*.

If Pierre is climbing uphill, then the graph is increasing:

If Pierre is going downhill, then the graph is decreasing:



**Example 1:**

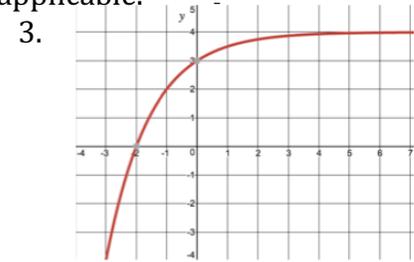
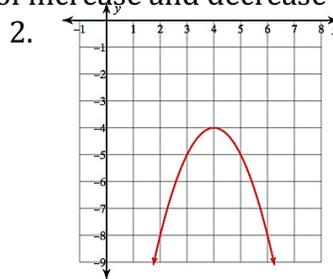
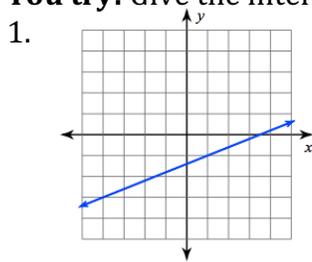


Write your Questions here!

**Intervals** of increasing and decreasing  
**ALWAYS** pertain to x-values.  
 Do **NOT** read numbers off the y-axis.  
**Stay on the x-axis for these intervals!**

Function Type	What Can you expect?
Linear	Will either increase or decrease for $\mathbb{R}$ depending on the slope. <ul style="list-style-type: none"> <li>Increasing for positive slope.</li> <li>Decreasing for negative slope.</li> </ul>
Quadratic	Will have a section of increase AND a section of decrease
Exponential	Will either increase or decrease for $\mathbb{R}$ depending on whether it is a growth or decay problem. <ul style="list-style-type: none"> <li>Increasing for growth.</li> <li>Decreasing for decay.</li> </ul>

**You try:** Give the intervals of increase and decrease as applicable.



Begin 4-5 Video 3

**The Tree Metaphor.**

It can get confusing when positive, negative, increasing and decreasing are all together. How to keep it straight? Think of a tree.

The parts of the tree that are \_\_\_\_\_ ground (x-axis) are **positive**.

The parts of the tree that are \_\_\_\_\_ ground are **negative**.

Branches growing up towards the \_\_\_\_\_? The height is **increasing**.

Branches turning back towards the \_\_\_\_\_? The height is **decreasing**.

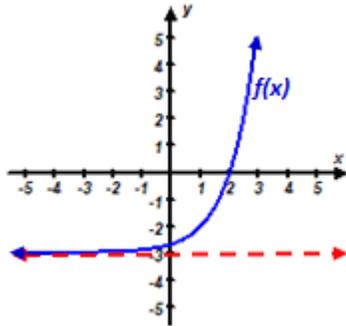


Write your Questions here!

# Practice Makes Better

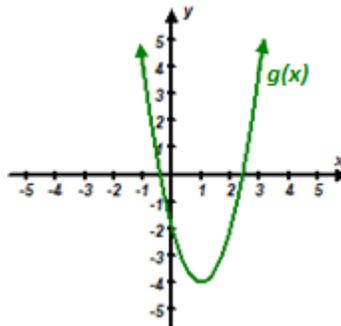
**Practice 4 – 5: Intervals of Positive & Negative/Increasing & Decreasing**  
 Find the intervals over which each function is positive and negative. Find the intervals over which each function is increasing and decreasing.

1.



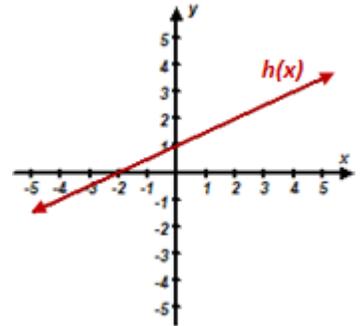
Positive:      Negative:  
 Increase:      Decrease:

2.



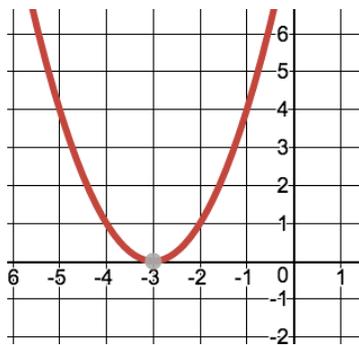
Positive:      Negative:  
 Increase:      Decrease:

3.



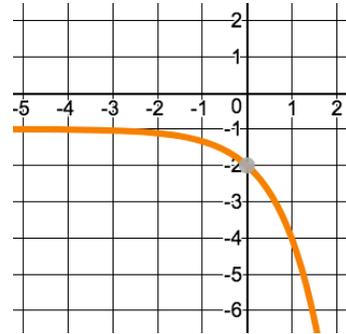
Positive:      Negative:  
 Increase:      Decrease:

4.



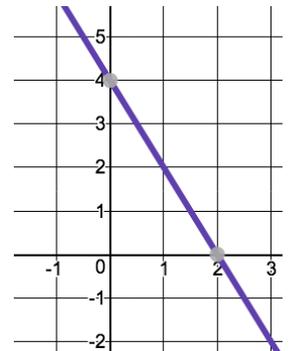
Positive:      Negative:  
 Increase:      Decrease:

5.



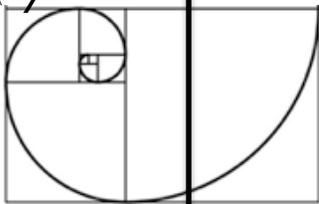
Positive:      Negative:  
 Increase:      Decrease:

6.



Positive:      Negative:  
 Increase:      Decrease:

## Spiral Practice

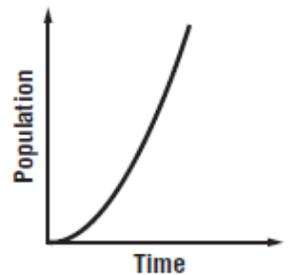


Change the following fractions into decimals, then tell whether the decimal is terminating or repeating

7.  $\frac{-13}{15}$

8.  $-9\frac{5}{8}$

9. The graph below shows the population of bacteria in a dish. Describe the change in population over time.



10. An oven is being preheated in order to bake a cake. Sketch a qualitative graph to represent the temperature of the oven over time.

Write your Questions here!

## 4-6: Special Characteristics

### Learning Targets:

- I can do math.  
F.IF.4

Begin 4-6 Video 1

**Vocabulary:**  
- Asymptote  
- Axis of Symmetry  
- Vertex

### Special Features of Quadratic Functions

#### VERTEX

The minimum or maximum point of a quadratic function



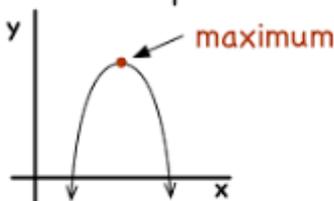
The vertex should always be written as an ordered pair.

A parabola that opens **upward** contains a vertex that is a \_\_\_\_\_ point.

A parabola that opens **downward** contains a vertex that is a \_\_\_\_\_ point.

#### MAXIMUM OF A QUADRATIC

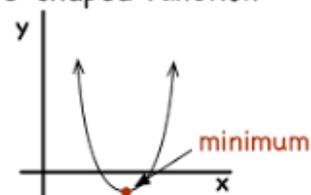
Highest point on the graph of an upside-down U shaped function



If the vertex is a *minimum*, then the range is all real numbers \_\_\_\_\_ than or equal to the *y*-value.

#### MINIMUM OF A QUADRATIC

Lowest point on the graph of a U-shaped function

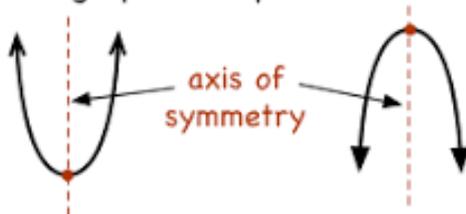


If the vertex is a *maximum*, then the range is all real numbers \_\_\_\_\_ than or equal to the *y*-value.

**We sometimes refer to the maximum and minimum as the extrema.**

#### AXIS OF SYMMETRY

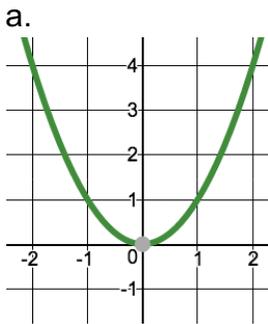
The vertical line through the vertex on the graph of a quadratic function



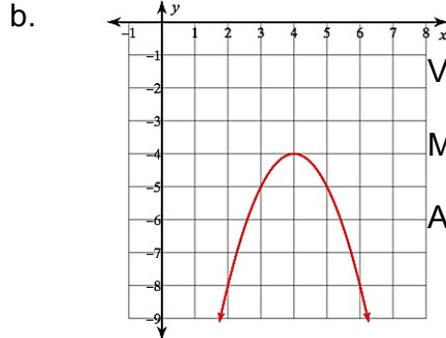
An **axis of symmetry** (also known as a **line of symmetry**) will divide the parabola into \_\_\_\_\_. The line of symmetry is always a \_\_\_\_\_ line of the form \_\_\_\_\_, where  $n$  is a real number. The axis of symmetry is often abbreviated as AOS.

Write your Questions here!

**Example 1:** Identify the vertex of each function. Is the vertex a minimum or maximum? Write an equation for the axis of symmetry.

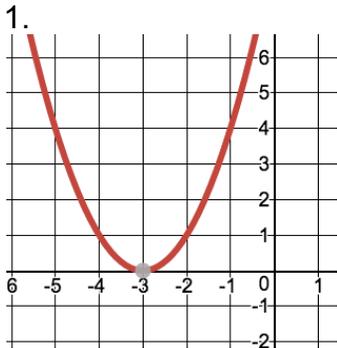


Vertex:  
Min or Max?  
Axis of Symmetry:

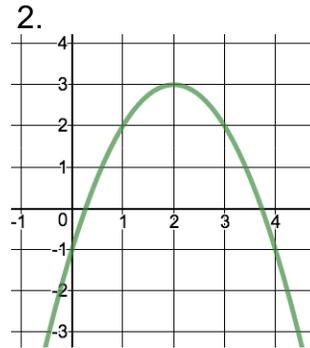


Vertex:  
Min or Max?  
Axis of Symmetry:

**You try:**



Vertex:  
Min or Max?  
Axis of Symmetry:



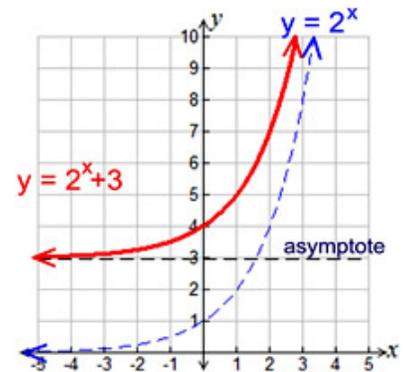
Vertex:  
Min or Max?  
Axis of Symmetry:



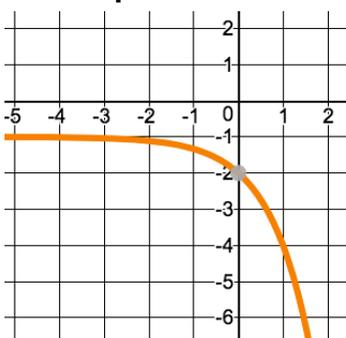
### Special Features of Exponential Functions

An asymptote is a line that a curve \_\_\_\_\_ (but does not touch) as it heads away from the origin.

- Every exponential function has a horizontal asymptote.
- Functions of the form  $y = a(b)^x$  (the exponential parent function) have horizontal asymptotes at \_\_\_\_\_ (the x-axis).
- Vertical shifts will shift the asymptote along with all the other points.
- Horizontal asymptotes should be written as an equation of the form \_\_\_\_\_, where  $c$  is the value of the vertical translation.

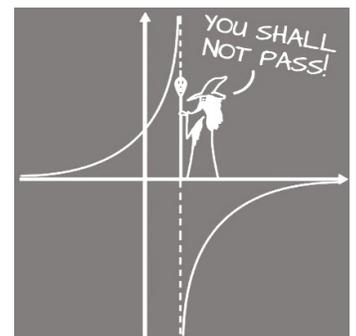


**Example 2:**



**Example 3:**

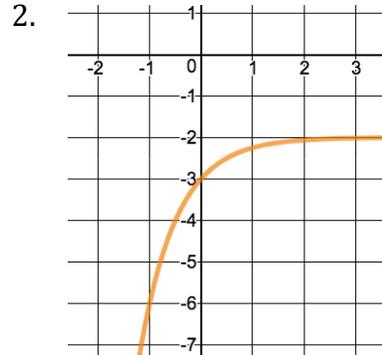
$$y = 3^x + 4$$



Write your Questions here!

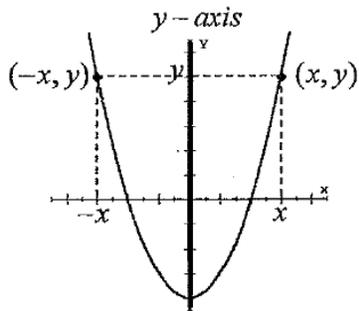
**You try:** Write the equation of the horizontal asymptote for the following functions.

1.  $y = 2^x + 1$



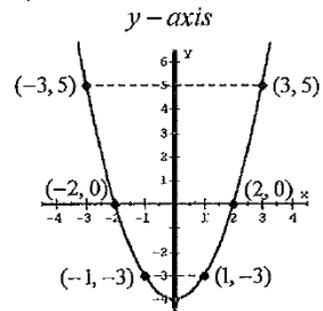
Begin 4-6 Video 3

Functions can be classified as Even, Odd or Neither.

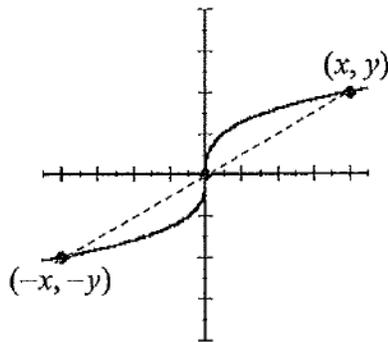


$f(-x) = f(x)$

**Even Functions** have symmetry about the y-axis.

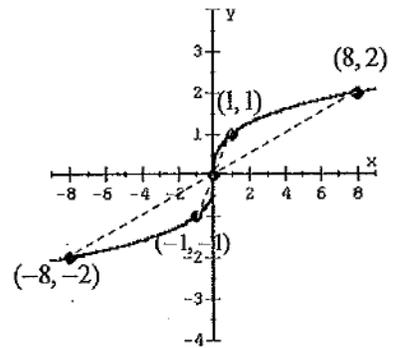


$y = x^2 - 4$



$f(-x) = -f(x)$

**Odd Functions** have symmetry about the origin.



$y = \sqrt[3]{x}$

Write your Questions here!

Test to determine whether a function is Even, Odd or Neither:  
Replace  $x$  with  $-x$  and compare the result to the original function.

- If  $f(-x) = f(x)$ , the function is \_\_\_\_\_.
- If  $f(-x) = -f(x)$ , the function is \_\_\_\_\_.
- If  $f(-x) \neq f(x)$  and  $f(-x) \neq -f(x)$ , the function is \_\_\_\_\_.

Type	What Can you expect?
Linear	<ul style="list-style-type: none"><li>• Sometimes Even<ul style="list-style-type: none"><li>○ Only horizontal lines</li><li>○ Of the form, <math>f(x) = b</math></li></ul></li><li>• Sometimes Odd<ul style="list-style-type: none"><li>○ Only lines that go through the origin</li><li>○ Of the form <math>f(x) = mx</math></li></ul></li><li>• Sometimes Neither</li></ul>
Quadratic	<ul style="list-style-type: none"><li>• Sometimes Even</li><li>• Sometimes Neither</li></ul>
Exponential	<ul style="list-style-type: none"><li>• Always Neither!</li></ul>

Hint: If you are getting lost in the  $x$ 's, try plugging in 2 and -2 to compare results.

**Example 4:**

$$f(x) = -3x^2$$

**Example 5:**

$$f(x) = -3x$$

**Example 6:**

$$f(x) = -3(2)^x$$

**You try:**

1.  $f(x) = (3)^x$

2.  $f(x) = x^2 + x + 6$

3.  $f(x) = x + 6$

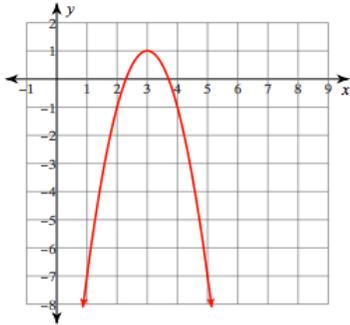
## Practice 4 – 6: Special Characteristics

Identify the vertex of each function. Is the vertex a minimum or maximum?

Write an equation for the axis of symmetry.

Write your Questions here!

1)  $y = -2x^2 + 12x - 17$

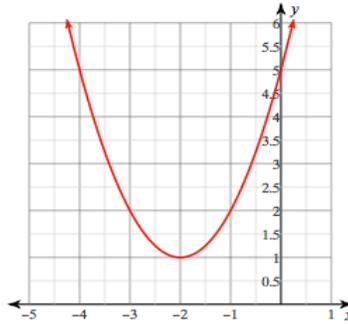


Vertex:

Min or Max?

Axis of Symmetry:

2)  $y = x^2 + 4x + 5$

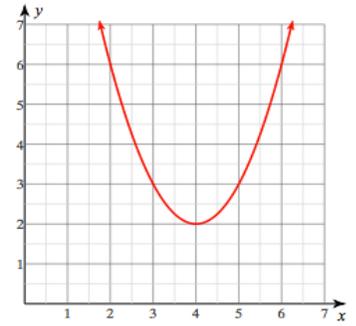


Vertex:

Min or Max?

Axis of Symmetry:

3)  $y = x^2 - 8x + 18$

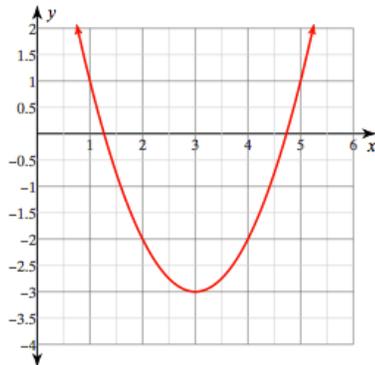


Vertex:

Min or Max?

Axis of Symmetry:

4)  $y = x^2 - 6x + 6$

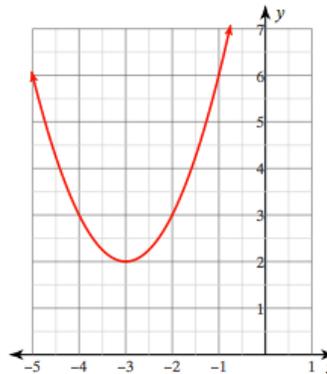


Vertex:

Min or Max?

Axis of Symmetry:

5)  $y = x^2 + 6x + 11$

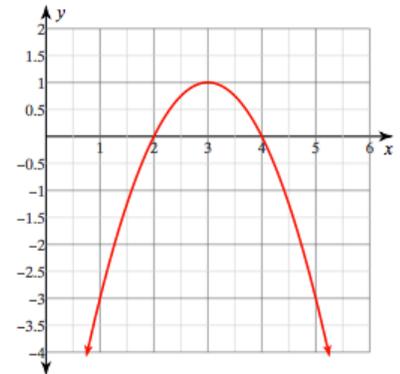


Vertex:

Min or Max?

Axis of Symmetry:

6)  $y = -x^2 + 6x - 8$



Vertex:

Min or Max?

Axis of Symmetry:

Write your Questions here!

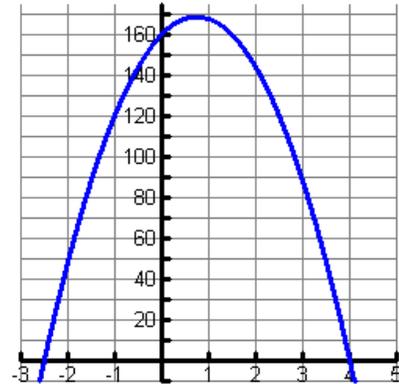
Wil E. Coyote is catapulting a boulder off a cliff to hit the road runner. Let  $t$  represent the number of seconds that the boulder catapults off the cliff and  $h(t)$  denote the height of the boulder, in feet, above the base of the cliff. Ignoring air resistance, we can use the following formula to express the path of the boulder:  $h(t) = -16t^2 + 24t + 160$

7. What does the x axis represent? \_\_\_\_\_ The y axis? \_\_\_\_\_

8. What part of the graph is insignificant? Why?

9. What was the height of the boulder before it was launched? What special point on the graph is associated with this information?

10. How long will it take before the boulder reaches the bottom of the cliff? What special point on the graph is associated with this information?



11. After how many seconds does the boulder change direction? \_\_\_\_\_

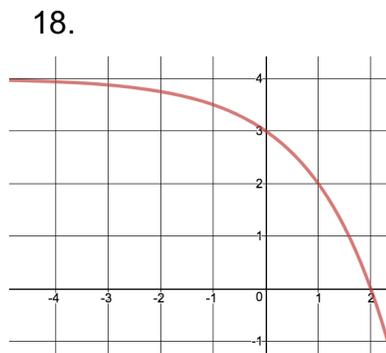
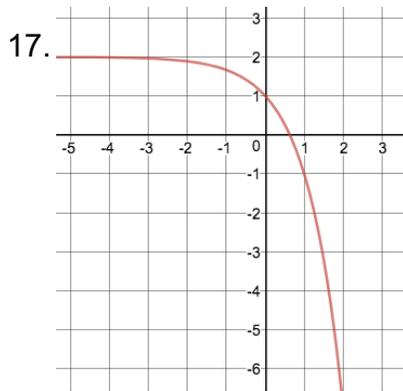
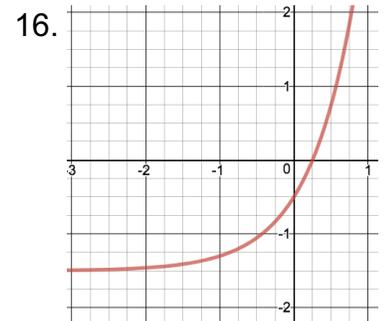
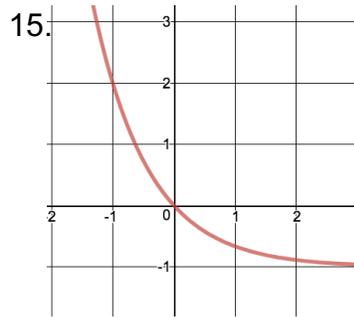
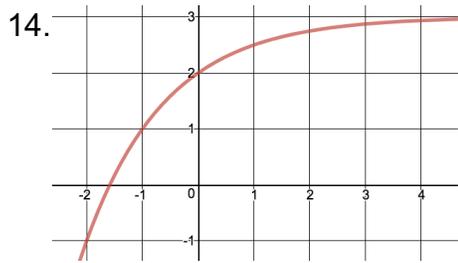
How high is the boulder when it changes direction? \_\_\_\_\_

What is this significant point called on the graph? \_\_\_\_\_

12. How high above the starting point does the boulder begin to change direction?

13. If Wil E. Coyote changes his mind, how many seconds does he have to stop the boulder from going over the cliff?

Write the equation of the horizontal asymptote for the following functions.



19.  $y = 3^x + 9$

20.  $f(x) = 3^x$

21.  $y = 3^x - 2.4$

Determine whether each of the following functions are even, odd or neither.

22.  $f(x) = 3^x$

23.  $y = 2x^2 + 3x - 9$

24.  $y = 2x + 9$

25.  $f(x) = -\left(\frac{1}{3}\right)^x$

26.  $y = 4x$

27.  $y = x^2 + 9$



Convert the decimals into fractions, make sure to reduce fractions to simplest form when possible and write improper fractions as mixed numbers

28. 0.4

29. 2.093

30. 0.324

31. A football club is hiring a painter to paint a mural on the concession stand wall. The painter charges an initial fee plus \$25 an hour. After 12 hours of work, the football club owed \$350. Create a function to represent the situation. Find and interpret the rate of change and initial value.

32. After writing part of his novel, Thomas is now writing consistent number of pages each week. The table below tracks Thomas' progress. Create a function to represent the situation. Find and interpret the rate of change.

Number of weeks	4	6	9
Pages Written	85	117	165

**Review your practice and notes to prepare for the mastery check.**

Write your Questions here!

## 4-7: End Behavior

### Learning Targets:

- I can identify the end behavior of a function.  
F.IF.4

Begin 4-7 Video 1

### Vocabulary:

- End Behaviors

One last characteristic that we can use to describe a function is end behavior. End behavior describes what happens to the \_\_\_\_\_ as the \_\_\_\_\_ approaches infinity or negative infinity.



### There are a few different ways to ask about end behavior:

As  $x \rightarrow -\infty$ ,  $y \rightarrow$  \_\_\_\_\_

(Read as x approaches negative infinity, y approaches \_\_\_\_?)

As  $x \rightarrow \infty$ ,  $y \rightarrow$  \_\_\_\_\_

(Read as x approaches positive infinity, y approaches \_\_\_\_?)

As  $x$  increases,  $y$  \_\_\_\_\_

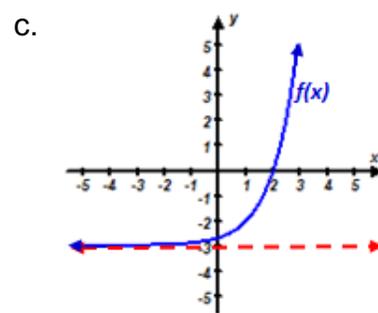
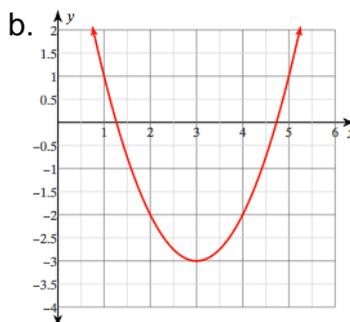
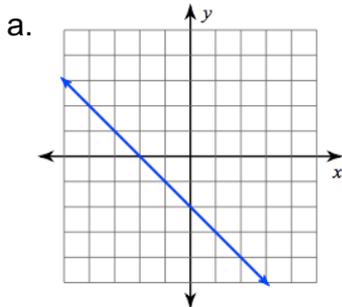
As  $x$  decreases,  $y$  \_\_\_\_\_

As  $x$  increases,  $f(x)$  \_\_\_\_\_

As  $x$  decreases,  $f(x)$  \_\_\_\_\_

Function Type	What Can you expect?
Linear	<ul style="list-style-type: none"> <li>One side will approach <math>\infty</math>.</li> <li>One side will approach <math>-\infty</math>.</li> </ul>
Quadratic	<ul style="list-style-type: none"> <li>Both sides will approach the same thing.</li> </ul>
Exponential	<ul style="list-style-type: none"> <li>One side will approach <math>\infty</math> or <math>-\infty</math>.</li> <li>One side will approach the asymptote.</li> </ul>

**Example 1:** Describe the end behavior of the functions below.



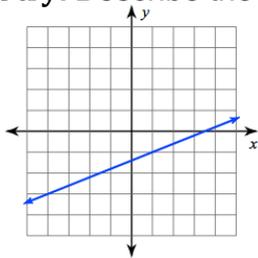
As  $x \rightarrow -\infty$ ,  $y \rightarrow$  \_\_\_\_\_ As  $x$  increases,  $y$  \_\_\_\_\_ As  $x$  decreases,  $f(x)$  \_\_\_\_\_

As  $x \rightarrow \infty$ ,  $y \rightarrow$  \_\_\_\_\_ As  $x$  decreases,  $y$  \_\_\_\_\_ As  $x$  increases,  $f(x)$  \_\_\_\_\_

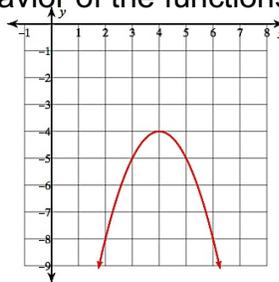
Write your Questions here!

**You try:** Describe the end behavior of the functions below.

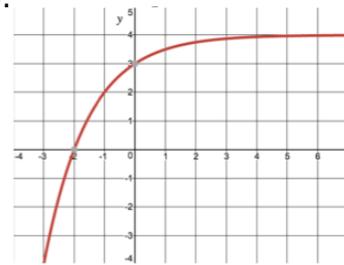
1.



2.



3.



As  $x \rightarrow -\infty$ ,  $y \rightarrow$  \_\_\_\_\_ As  $x$  increases,  $y$  \_\_\_\_\_ As  $x$  decreases,  $f(x)$  \_\_\_\_\_

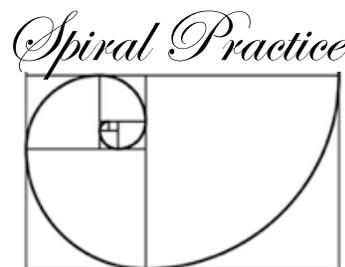
As  $x \rightarrow \infty$ ,  $y \rightarrow$  \_\_\_\_\_ As  $x$  decreases,  $y$  \_\_\_\_\_ As  $x$  increases,  $f(x)$  \_\_\_\_\_

## Practice Makes Better

### Practice 4 – 7: End Behavior

**Surprise! We are going to do spirals first this time.**

1. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 3 and 4. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.



This WILL be on your mastery check!

Raymond's Solution:

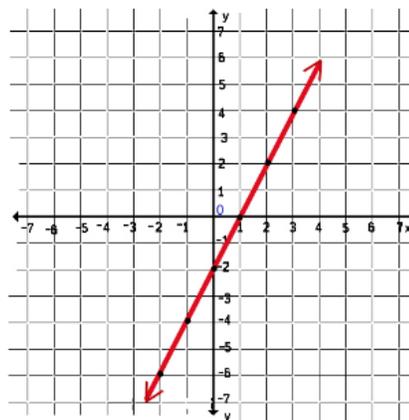
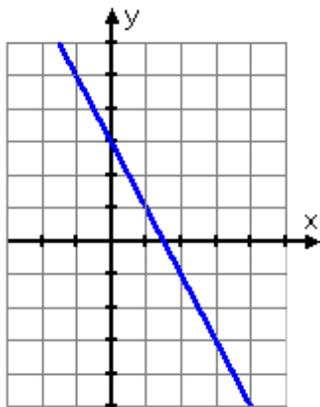
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 3^2 &= c^2 \\ 16 + 9 &= c^2 \\ 25 &= c \end{aligned}$$

Correct Solution:

2. Evaluate the expressions when  $a = 156$  and  $b = 12$   
 $\sqrt{a-b}$

3. Marcus told his mother that the area of the square office at his school is  $125 \text{ ft}^2$ . His mom asked him for the length of one side of the office. What is the length of the side of the office?

Summarize the characteristics of the following functions.



a. rate of change (slope)

a. rate of change (slope)

b. domain

c. range

b. domain

c. range

d. x-intercept

e. y-intercept

d. x-intercept

e. y-intercept

f. interval of inc.

g. interval of dec.

f. interval of inc.

g. interval of dec.

h. interval of pos.

i. interval of neg.

h. interval of pos.

i. interval of neg.

j. End Behavior:

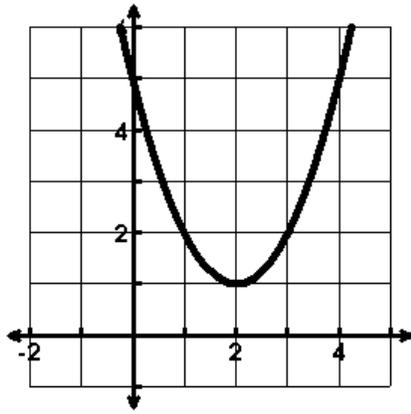
as  $x \rightarrow -\infty, y \rightarrow$  \_\_\_\_\_

as  $x \rightarrow \infty, y \rightarrow$  \_\_\_\_\_

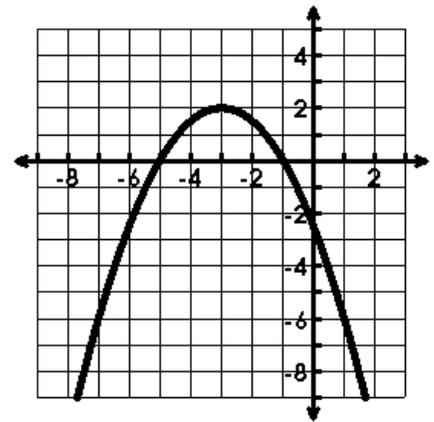
j. End Behavior:

as  $x \rightarrow -\infty, y \rightarrow$  \_\_\_\_\_

as  $x \rightarrow \infty, y \rightarrow$  \_\_\_\_\_

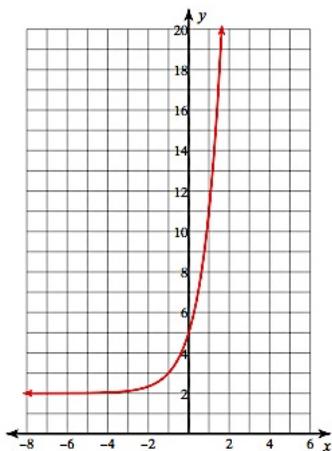


- a. Axis of Symmetry: \_\_\_\_\_
- b. Vertex: \_\_\_\_\_
- Max: \_\_\_\_\_ or Min: \_\_\_\_\_
- c. x-intercept  
\_\_\_\_\_, \_\_\_\_\_
- d. Y-intercept: \_\_\_\_\_
- e. Domain: \_\_\_\_\_
- f. Range: \_\_\_\_\_
- g. Interval of  
Increasing: \_\_\_\_\_
- h. Interval of  
Decreasing: \_\_\_\_\_
- i. Interval of  
Positive: \_\_\_\_\_
- j. Interval of  
Negative: \_\_\_\_\_
- k. Rate of Change over  
[1,3]: \_\_\_\_\_
- l. End Behavior:  
As  $x \rightarrow -\infty, y \rightarrow$  \_\_\_\_\_  
As  $x \rightarrow \infty, y \rightarrow$  \_\_\_\_\_



- a. AOS: \_\_\_\_\_
- b. Vertex: \_\_\_\_\_
- Max: \_\_\_\_\_ or Min: \_\_\_\_\_
- c. x-intercept(s): \_\_\_\_\_, \_\_\_\_\_
- d. Y-intercept: \_\_\_\_\_
- e. Domain: \_\_\_\_\_
- f. Range: \_\_\_\_\_
- g. Interval of  
Increasing: \_\_\_\_\_
- h. Interval of  
Decreasing: \_\_\_\_\_
- i. Interval of  
Positive: \_\_\_\_\_
- j. Interval of  
Negative: \_\_\_\_\_
- k. Rate of Change over  
[-7,0]: \_\_\_\_\_
- l. End Behavior:  
As  $x \rightarrow -\infty, y \rightarrow$  \_\_\_\_\_  
As  $x \rightarrow \infty, y \rightarrow$  \_\_\_\_\_

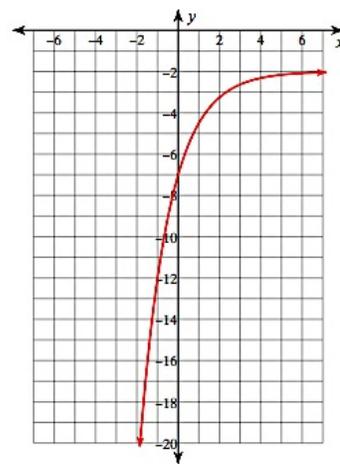
Write your Questions here!



- Asymptote: \_\_\_\_\_
- X-intercept: \_\_\_\_\_
- Y-intercept: \_\_\_\_\_
- Domain: \_\_\_\_\_
- Range: \_\_\_\_\_
- Interval of Increasing: \_\_\_\_\_
- Interval of Decreasing: \_\_\_\_\_
- Interval of Positive: \_\_\_\_\_
- Interval of Negative: \_\_\_\_\_
- Rate of Change over  $[-1, 0]$ : \_\_\_\_\_
- End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$$

$$\text{As } x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$$



- Asymptote: \_\_\_\_\_
- X-intercept: \_\_\_\_\_
- Y-intercept: \_\_\_\_\_
- Domain: \_\_\_\_\_
- Range: \_\_\_\_\_
- Interval of Increasing: \_\_\_\_\_
- Interval of Decreasing: \_\_\_\_\_
- Interval of Positive: \_\_\_\_\_
- Interval of Negative: \_\_\_\_\_
- Rate of Change over  $[-1, 0]$ : \_\_\_\_\_
- End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$$

$$\text{As } x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$$

**Review your practice and notes to prepare for the mastery check.**



## Unit 4 Study Guide

For the questions 1-3, tell whether the following scenarios describe a linear, exponential, or quadratic function.

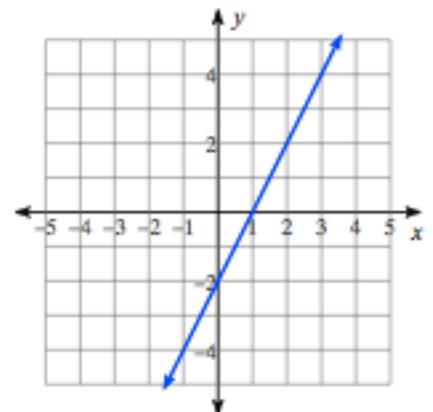
- Given a function exhibits the following end behavior:  
As  $x$  decreases,  $f(x)$  increases  
As  $x$  increases,  $f(x)$  increases
- Given a function exhibits the following end behavior:  
As  $x$  decreases,  $f(x)$  increases  
As  $x$  increases,  $f(x)$  approaches 4
- Given a function exhibits the following end behavior:  
As  $x$  decreases,  $f(x)$  increases  
As  $x$  increases,  $f(x)$  decrease
- A machine makes bolts at an average of 250 per hour for up to 10 hours. Let  $y$  be the number of bolts that the machine can make for a given  $x$  amount of time. What is the domain of the function? What is the range of the function?
- Marco opened a savings account with \$70. Each month, he plans to save \$40. Write a function that represents this situation. What is the domain of the function? What does the domain represent?

**6-10. Use the graph shown below to answer the following questions.**

- What is the domain of the function?
- What is the range of the function?
- What is the  $y$ -intercept of the function?
- What is the  $x$  intercept of the function?
- When  $x > 1$ , the function is \_\_\_\_\_ and \_\_\_\_\_.
- Find the average rate of change of  $f(x)$  over the following intervals.

A.  $-1 \leq x \leq 2$

B.  $2 \leq x \leq 4$



$x$	$f(x)$
-1	17
0	20
1	23
2	26
4	32

What is the asymptote of the exponential functions?

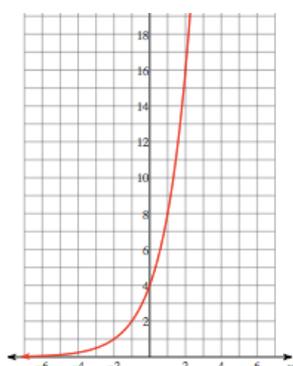
12.  $f(x) = -3(2)^x$       13.  $f(x) = 2(1/2)^x + 1$       14.  $f(x) = 4(5)^x - 3$

15. What is the average rate of change for the functions  $f(x)$  from  $x = 1$  and  $x = 3$ ?

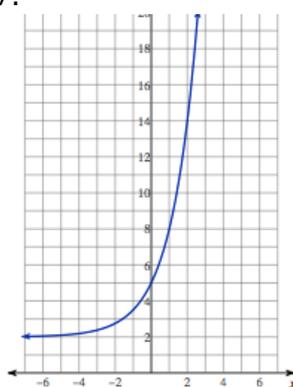
$$f(x) = -3^x + 2$$

For each of the following functions describe the interval of increase and/or decrease.

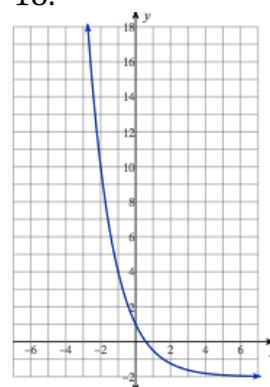
16.



17.



18.



19. For which of the above graph is the following statement true. “as  $x$  decreases,  $f(x)$  approaches 0”

20. For which of the above graphs is the following statement true. “as  $x$  increases,  $f(x)$  approaches -2”

21. For which of the above graphs is the following statement true. “as  $x$  increases,  $f(x)$  approaches infinity”

22. What is the asymptote for 16 \_\_\_\_\_ for 17 \_\_\_\_\_ and 18 \_\_\_\_\_?

23. Which has a greater rate of change from  $x = 0$  to  $x = 4$ ,  $f(x)$  or  $g(x)$ ?

$$f(x) = 3^x + 1$$

x	g(x)
0	2
1	16
2	25
3	50
4	100

**Find the y-intercept. (To find the y-intercept replace  $x = 0$  and solve for  $y$ .)**

24.  $f(x) = 4^x$

25.  $f(x) = 2^x - 1$

26.  $f(x) = 3(2)^x + 4$

27.  $f(x) = 3x - 6$

28.  $f(x) = 4x - 2$

29. Busy Golf: Rents golf carts for \$35 plus \$7 per hour.  
Cruising Golf rental fees are as follows:

Hours ( $x$ )	Total Cost $g(x)$
2	34
3	44
4	54
5	64
6	74
7	84

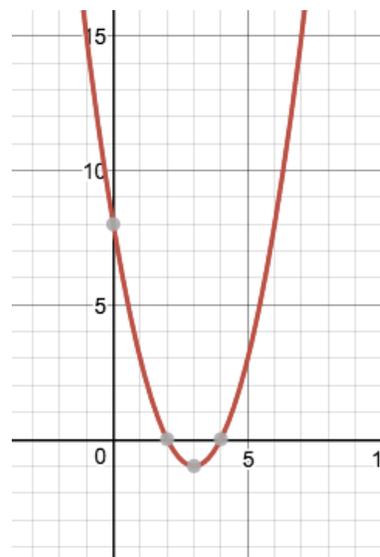
Find and compare the y-intercepts of the two functions.

Find and compare the rate for change for each function.

Based on this information, which rental would you choose? Write a short paragraph explaining why you think Busy Golf or Cruising Golf would be the better choice. Be sure to support your decision with mathematical reasoning.

30.

- a. Domain \_\_\_\_\_
- b. Range \_\_\_\_\_
- c. Interval of Increase: \_\_\_\_\_
- d. Interval of Decrease: \_\_\_\_\_
- e. What is the end behavior of the graph?  
*as  $x \rightarrow \infty$  (increases),  $f(x) \rightarrow$  \_\_\_\_\_*  
*As  $x \rightarrow -\infty$  (decreases),  $f(x) \rightarrow$  \_\_\_\_\_*
- f. y-intercept: \_\_\_\_\_
- g. x-intercept(s): \_\_\_\_\_
- h. What are the coordinates of the vertex?  
 \_\_\_\_\_
- i. Give an equation for the Axis of Symmetry.  
 \_\_\_\_\_
- j. Is the extreme point a maximum or a minimum?  
 \_\_\_\_\_
- k. What is the rate of change of this function over the interval [1,3]? \_\_\_\_\_



31. Check all the boxes that apply to the given functions as transformations to the parent function.

	Horizontal Shift Left	Horizontal Shift Right	Vertical Shift Up	Vertical Shift Down	Vertical Stretch	Vertical Shrink	Reflect
$f(x) = -4 \cdot 2^x$							
$g(x) = 2(x - 5)^2 - 1$							
$h(x) = -(1/2)x + 4$							

32. For each equation below, write a new equation that shift the function down 6 and reflects over the x axis.

a.  $f(x) = 3^x$

b.  $g(x) = x^2$

## Glossary

**Average Rate of Change:** The change in the value of a quantity by the elapsed time. For a function, this is the change in the  $y$ -value divided by the change in the  $x$ -value for two distinct points on the graph.

**Coefficient:** A number multiplied by a variable in an algebraic expression.

**Constant Rate of Change:** With respect to the variable  $x$  of a linear function  $y = f(x)$ , the constant rate of change is the slope of its graph.

**Constant Term:** A quantity that does not change its value.

**Coordinate Plane:** The plane determined by a horizontal number line, called the  $x$ -axis, and a vertical number line, called the  $y$ -axis, intersecting at a point called the origin. Each point in the coordinate plane can be specified by an ordered pair of numbers.

**Continuous:** Describes a connected set of numbers, such as an interval.

**Discrete:** A set with elements that are disconnected.

**Domain:** The set of  $x$ -coordinates of the set of points on a graph; the set of  $x$ -coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

**End Behaviors:** The appearance of a graph as it is followed farther and farther in either direction.

**Exponential Function:** A nonlinear function in which the independent value is an exponent in the function, as in  $y = ab^x$ .

**Exponential Model:** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

**Horizontal Shift:** A rigid transformation of a graph in a horizontal direction, either left or right.

**Interval Notation:** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or

brackets are used to show whether the endpoints are excluded or included.

**Linear Function:** A function with a constant rate of change and a straight line graph.

**Linear Model:** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

**Parameter:** The independent variable or variables in a system of equations with more than one dependent variable.

**Quadratic Equation:** An equation of degree 2, which has at most two solutions.

**Quadratic Function:** A function of degree 2 which has a graph that "turns around" once, resembling an umbrella-like curve that faces either right-side up or upside down. This graph is called a parabola.

**Range:** The set of all possible outputs of a function.

**Reflection:** A transformation that "flips" a figure over a mirror or reflection line.

**Root:** The  $x$ -values where the function has a value of zero.

**Slope:** The ratio of the vertical and horizontal changes between two points on a surface or a line.

**Variable:** A letter or symbol used to represent a number.

**Vertex:** The maximum or minimum value of a parabola, either in terms of  $y$  if the parabola is opening up or down, or in terms of  $x$  if the parabola is opening left or right.

**Vertical Translation:** A shift in which a plane figure moves vertically.

**X-intercept:** The point where a line meets or crosses the  $x$ -axis.  $(x,0)$

**Y-Intercept:** The point where a line meets or crosses the  $y$ -axis.  $(0,y)$